EXPONENTIAL STABILITY OF HOPFIELD CONFORMABLE FRACTIONAL-ORDER POLYTOPIC NEURAL NETWORKS

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ABSTRACT

Due to many reasons such as linear approximation, external noises, modeling inaccuracies, measurement errors, and so on, uncertain disturbances are usually unavoidable in real dynamical systems. Convex polytopic uncertainties are one of a kind of these disturbances. In this paper, we consider the problem of fractional exponential stability for a class of Hopfield fractional-order neural networks (FONNs) subject to conformable derivative and convex polytopic uncertainties. By using the fractional Lyapunov functional method combined with some calculations on matrices, a new sufficient condition on fractional exponential stability for conformable FONNs is established via linear matrix inequalities (LMIs), which therefore can be efficiently solved in polynomial time by using the existing convex algorithms. The proposed result is quite general and improves those given in the literature since many factors such as conformable fractional derivative, convex polytopic uncertainties, exponential stability, are considered. A numerical example is provided to demonstrate the correctness of the theoretical results.

TÍNH ỔN ĐỊNH MŨ CỦA MẠNG NƠ RON PHÂN THỨ HOPFIELD PHÙ HỢP TỔ HỢP LỒI

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TỪ KHÓA

Mạng nơ ron Hopfiled phân thứ
Định lý Lyapunov phân thứ
Nhiễu tổ hợp lồi
Ổn định mũ
Bất đẳng thức ma trận tuyến tính

TÓM TẮT

Nhiễu thường xuyên xuất hiện trong các hệ động lực trong thực tế bởi nhiều nguyên nhân như quá trình xấp xỉ tuyến tính, lỗi do đo đạc, lỗi trong quá trình mô hình hóa. Nhiễu dạng tổ hợp lồi là một trong những loại nhiễu này. Trong bài báo này, chúng tôi nghiên cứu tính ổn định mũ cho một lớp mạng nơ ron Hopfield phân thứ phù hợp với nhiễu dạng tổ hợp lồi. Bằng cách sử dụng phương pháp hàm Lyapunov cho hệ phương trình vi phân phân thứ kết hợp với một số phép biến đổi trên ma trận, một điều kiện đủ cho tính ổn định mũ của mạng nơ ron Hopfiled phân thứ phù hợp được thiết lập dưới dạng bất đẳng thức ma trận tuyến tính. Điều kiện này có thể giải hiệu quả trong thời gian đa thức bởi các thuật toán tối ưu lồi. Các điều kiện được đưa ra ở đây tổng quát và cải tiến so với một số kết quả đã có bởi vì một số yếu tố như đạo hàm phân thứ phù hợp, nhiễu dạng tổ hợp lồi, tính ổn định mũ đã được xét đến. Một ví dụ số được đưa ra để minh họa cho tính chính xác của kết quả lý thuyết thu được.

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1. Introduction

The interest in fractional-order neural networks (FONNs) has grown rapidly due to their successful applications in different areas such as mathematical modeling, pattern recognition, and signal processing [1]-[4].

Investigating the stability analysis of FONNs is one of the important problems and many interesting results have been published in the literature [5]-[9]. With the help of the fractional-order Lyapunov direct method, the authors in [5] derived stability conditions in terms of LMIs for Caputo FONNs. The results in [5] were extended to Caputo FONNs with time delays by Y. Yang et al. [6]. Using the S-procedure technique and fractional Razumikhin-type theorem, the authors in [7] proposed an LMI-based stability condition for delayed Caputo FONNs. The problem of stability analysis for some kinds of FONNs such as complex-valued projective FONNs, and neutral type memristor-based FONNs have been considered in [8] and [9], respectively. It should be noted that almost all of the existing results on the problem are focused on Caputo FONNs or Riemann-Liouville FONNs (see [5]-[9] and references therein), and very few works are devoted to conformable FONNs [10], [11]. With the help of the Lyapunov functional method, the authors in [10] considered existence, uniqueness, and exponential stability problems for Hopfield FONNs subject to conformable fractional derivatives. Note that their results are in terms of matrices elements, which cannot propose the condition in terms of the whole matrix. Recently, the authors in [11] derived some conditions to guarantee stability analysis of Hopfield conformable FONNs subject to time-varying parametric perturbations. The conditions are in terms LMIs that are numerically tractable. It is worth noticing that the convex polytopic uncertainties are not considered in the model of the paper [10], [11]. To the best of our knowledge, the problem of fractional exponential stability for conformable FONNs with convex polytope uncertainties has not yet been addressed in the literature.

In this paper, we present a novel approach to study the problem of fractional exponential stability of Hopfield conformable FONNs with convex polytopic uncertainties. Our approach is based on using conformable fractional-order Lyapunov theorem and LMIs techniques. Consequently, a new criterion for the problem is established. Moreover, a numerical example is given to show that our results are less conservative than the results in [11].

Notations: A matrix $\mathbf{P}$ is symmetric positive definite, write $\mathbf{P} > 0,$ if $\mathbf{P} = \mathbf{P}^T$, and $y^T\mathbf{P}y > 0,$ for all $y \in \mathbb{R}^n, y \neq 0.$ $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues respectively. Let S^+ and S^{++} stand for the set of symmetric semi-positive definite matrix and symmetric positive definite matrices in $\mathbb{R}^{n\times n}$, respectively.

2. Preliminaries and Problem statement

First, we recal definition of conformable fractional derivative [1].

Definition 1 [12] Let a function $g:[0,+\infty)\to\mathbb{R},$ the conformable fractional derivative of the function g of order $\alpha\in(0,1)$ is defined by $T^\alpha g(t)=\lim\limits_{\varepsilon\to 0}\dfrac{g\left(t+\varepsilon t^{1-\alpha}\right)-g(t)}{\varepsilon}, \forall t>0.$ If $T^\alpha g(t)$ exists on $(0,+\infty),$ then $T^\alpha g(0)=\lim\limits_{t\to 0^+}T^\alpha g(t).$ If the conformable fractional derivative $g(t)$ of order α exists on $(0,+\infty),$ then the function $g(t)$ is said to be $\alpha-$ differentiable on the interval $(0,+\infty).$

For a vector function $x(t)=(x_1(t),\ldots,x_n(t))^T \in \mathbb{R}^n$, the conformable fractional derivative of $x(t)$ is defined for each component as follows

$T^\alpha x(t) := (T^\alpha x_1(t),\ldots,T^\alpha x_n(t))^T$.

P1 [12]: For any scalars $a,b\in\mathbb{R}$, and two functions $f_1, f_2 : [0,+\infty)\to\mathbb{R}$, we have

$T^\alpha (af_1(t)+bf_2(t)) = aT^\alpha f_1(t)+bT^\alpha f_2(t), \forall t\ge 0, 0<\alpha\le 1.$

P2 [13]: Let $y:[0,+\infty)\to\mathbb{R}^n$ such that $T^\alpha y(t)$ exists on [0, ∞) and $\mathbf{R}\in S^{++}$. Then, we have $T^\alpha y^T(t)\mathbf{R}y(t)$ exists on [0, ∞) and

$T^\alpha y^T(t)\mathbf{R}y(t) = 2y^T(t)\mathbf{R}T^\alpha y(t), \forall t\ge 0, 0<\alpha\le 1.$

Consider the following Hopfield conformable fractional order polytopic neural networks (NNs)

$$\begin{cases} T^\alpha y(t) = -\mathbf{A}(\xi)y(t)+\mathbf{W}(\xi)g(y(t)), t\ge 0 \\ y(0)=y_0, \end{cases} \tag{1}$$

where $\alpha\in(0,1]$ is the order of system (1), $y(t)=(y_1(t),\ldots,y_n(t))\in\mathbb{R}^n$ is the state vector, $g(y(t))=(g_1(y_1(t)),\ldots,g_n(y_n(t)))\in\mathbb{R}^n$ stand for the neuron activation function of the networks, $y_0\in\mathbb{R}^n$ is the initial condition. The system matrices $\{\mathbf{A}(\xi),\mathbf{W}(\xi)\}$ are belong to a polytope Ω given by

$$\Omega = \left\{[\mathbf{A},\mathbf{W}](\xi) := \sum_{i=1}^{N}\xi_i[\mathbf{A}_i,\mathbf{W}_i], \sum_{i=1}^{N}\xi_i = 1, \xi_i\ge 0\right\},$$

with vertices $\{\mathbf{A}_i,\mathbf{W}_i\}$, where $\mathbf{A}_i = diag\{a_1^i,\ldots,a_n^i\}\in\mathbb{R}^n$ $(a_k^i>0, \forall k=1,\ldots,n, i=1,\ldots,N)$ are given diagonal matrices, $\mathbf{W}_i\in\mathbb{R}^n$ $(i=1,\ldots,N)$ are given constant matrices, parameters ξ_i $(i=1,\ldots,N)$ are time-invariant. The functions $g_j(.)$ are continuous, $g_j(0)=0$, $(j=1,\ldots,n)$, and Lipschitz condition on $\mathbb{R}$ with Lipschitz constants $\kappa_j>0$:

$$|g_j(a)-g_j(b)|\le\kappa_j|a-b|, \forall a,b\in\mathbb{R}, j=1,\ldots,n. \tag{2}$$

Definition 2 [13] System (1) is said to be fractional exponentially stable if

$\|y(t)\|\le K\|y_0\|e^{-\beta\frac{t^\alpha}{\alpha}}, \forall t\ge 0, 0<\alpha\le 1.$

Let us recall the following useful well-known lemma.

Lemma 1 [13] The system (1) is fractional exponentially stable if there exist $\theta_k>0 (k=1,2,3)$, and a continuous function $V:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}$ such that the following conditions hold

$(i)\ \theta_1\|y\|^2\le V(t,y)\le\theta_2\|y\|^2$,

$(ii)\ V(t,y(t))$ is $\alpha-$differentiable on the interval $(0,+\infty)$,

$(iii)\ T^\alpha V(t,y)\le-\theta_3\|y\|^2$.

3. Main Results

Let $\mathbf{S}\in \mathrm{S}^{+},\mathbf{P}_i\in \mathrm{S}^{++}\left(i=1,\dots,N\right)$, we denote $\mathbf{L}=diag\left\{\kappa_1,\dots,\kappa_n\right\}$,

$$\mathbf{P}\left(\xi\right)=\sum_{i=1}^{N}\xi_i\mathbf{P}_{\mathbf{i}},S=\begin{bmatrix}\mathbf{S} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},$$

$$\Psi_i\left(\mathbf{A}_i,\mathbf{W}_i,\mathbf{P}_j\right)=\begin{bmatrix}-\mathbf{A}_i^T\mathbf{P}_j-\mathbf{P}_j\mathbf{A}_i+\varepsilon\mathbf{L}^T\mathbf{L} & \mathbf{P}_j\mathbf{W}_i\\ \mathbf{W}_i^T\mathbf{P}_j & -\varepsilon\mathbf{I}\end{bmatrix},$$

where $\kappa_i\left(i=1,\dots,n\right)$ are Lipschitz constants, other scalars and matrices are defined as in Section 2.

Theorem 1 *The system* (1) *is fractional exponentially stable if there exist* $\mathbf{S}\in \mathrm{S}^{+},\mathbf{P}_i\in \mathrm{S}^{++}\left(i=1,\dots,N\right)$, *and a scalar* $\varepsilon>0$ *such that the following conditions hold*:

$$\Psi_i\left(\mathbf{A}_i,\mathbf{W}_i,\mathbf{P}_i\right)<-S,i=1,2,\dots,N, \tag{3}$$

$$\Psi_i\left(\mathbf{A}_i,\mathbf{W}_i,\mathbf{P}_j\right)+\Psi_j\left(\mathbf{A}_j,\mathbf{W}_j,\mathbf{P}_i\right)<\frac{2}{N-1}\mathbf{S},i=1,\dots,N-1,\ j=i+1,\dots,N. \tag{4}$$

Proof. Let us consider the following Lyapunov function

$V\left(t\right)=V\left(t,y\left(t\right)\right)=y^T\left(t\right)\mathbf{P}\left(\xi\right)y\left(t\right),t\geq 0.$

It is clear that

$\Gamma_1\left\|y\left(t\right)\right\|^2\leq V\left(t,y\left(t\right)\right)\leq \Gamma_2\left\|y\left(t\right)\right\|^2,\forall t\geq 0,$

where $\Gamma_1=\min\limits_{i=1,\dots,N}\left\{\lambda_{\min}\left(\mathbf{P}_i\right)\right\},\Gamma_2=\max\limits_{i=1,\dots,N}\left\{\lambda_{\max}\left(\mathbf{P}_i\right)\right\}$. So condition (i) in Lemma 1 is guaranteed. Using property P2, we calculate the $\alpha-$order conformable derivative of $V\left(t\right)$ along the trajectories of the system (1) as follows:

$$\begin{aligned}T^{\alpha}V\left(t\right)&=2y^T\left(t\right)\mathbf{P}\left(\xi\right)T^{\alpha}y\left(t\right)\\ &=y^T\left(t\right)\left[-\mathbf{P}\left(\xi\right)\mathbf{A}\left(\xi\right)-\mathbf{A}^T\left(\xi\right)\mathbf{P}\left(\xi\right)\right]y\left(t\right)\\ &\quad+2y^T\left(t\right)\mathbf{P}\left(\xi\right)\mathbf{W}\left(\xi\right)g\left(y\left(t\right)\right).\end{aligned} \tag{5}$$

With the help of Cauchy matrix inequality and condition (2), we obtain

$$\begin{aligned}&2y^T\left(t\right)\mathbf{P}\left(\xi\right)\mathbf{W}\left(\xi\right)g\left(y\left(t\right)\right)\\ &\leq \varepsilon^{-1}y^T\left(t\right)\mathbf{P}\left(\xi\right)\mathbf{W}\left(\xi\right)\mathbf{W}^T\left(\xi\right)\mathbf{P}\left(\xi\right)y\left(t\right)+\varepsilon g^T\left(y\left(t\right)\right)g\left(y\left(t\right)\right)\\ &\leq \varepsilon^{-1}y^T\left(t\right)\mathbf{P}\left(\xi\right)\mathbf{W}\left(\xi\right)\mathbf{W}^T\left(\xi\right)\mathbf{P}\left(\xi\right)y\left(t\right)+\varepsilon y^T\left(t\right)\mathbf{L}^T\mathbf{L}y\left(t\right).\end{aligned} \tag{6}$$

From (5) and (6), we have

$T^{\alpha}V\left(t\right)\leq y^T\left(t\right)\Omega\left(\xi\right)y\left(t\right),$

where

$\Omega\left(\xi\right)=-\mathbf{P}\left(\xi\right)\mathbf{A}\left(\xi\right)-\mathbf{A}^T\left(\xi\right)\mathbf{P}\left(\xi\right)+\varepsilon^{-1}\mathbf{P}\left(\xi\right)\mathbf{W}\left(\xi\right)\mathbf{W}^T\left(\xi\right)\mathbf{P}\left(\xi\right)+\varepsilon\mathbf{L}^T\mathbf{L}.$

Hence

$$T^{\alpha}V\left(t\right)\leq \lambda_{\max}\left(\Omega\left(\xi\right)\right)\left\|y\left(t\right)\right\|^2,\forall t\geq 0. \tag{7}$$

Using Schur Complement Lemma [14], $\Omega\left(\xi\right)<0$, if

$$H(\xi)=\begin{bmatrix} H_{11}(\xi) & \mathbf{P}(\xi)\mathbf{W}(\xi) \\ \mathbf{W}^T(\xi)\mathbf{P}(\xi) & -\varepsilon\mathbf{I} \end{bmatrix}<0,$$

where $H_{11}(\xi)=-\mathbf{P}(\xi)\mathbf{A}(\xi)-\mathbf{A}^T(\xi)\mathbf{P}(\xi).$

Since $\mathbf{P}(\xi)=\sum_{i=1}^{N}\xi_i\mathbf{P}_i, \mathbf{A}(\xi)=\sum_{i=1}^{N}\xi_i\mathbf{A}_i, \mathbf{W}(\xi)=\sum_{i=1}^{N}\xi_i\mathbf{W}_i, \sum_{i=1}^{N}\xi_i=1, \xi_i\ge 0,$ we have

$$H(\xi)=\sum_{i=1}^{N}\xi_i^2\Psi_i(\mathbf{A}_i,\mathbf{W}_i,\mathbf{P}_i)+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_i\xi_j\left[\Psi_i(\mathbf{A}_i,\mathbf{W}_i,\mathbf{P}_j)+\Psi_j(\mathbf{A}_j,\mathbf{W}_j,\mathbf{P}_i)\right].$$

It follows from (3) and (4) that

$$H(\xi)\le-\sum_{i=1}^{N}\xi_i^2S+\frac{2}{N-1}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_i\xi_jS=\left[-\sum_{i=1}^{N}\xi_i^2+\frac{2}{N-1}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_i\xi_j\right]S.$$

From the relation

$$(N-1)\sum_{i=1}^{N}\xi_i^2-2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_i\xi_j=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(\xi_i-\xi_j)^2\ge 0,$$

we have

$$\left[-\sum_{i=1}^{N}\xi_i^2+\frac{2}{N-1}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_i\xi_j\right]S<0,$$

which implies that $\Omega(\xi)<0$ provided the conditions (3) and (4) hold. Since $\Omega(\xi)<0,$ there exists a scalar $\theta>0$ such that $T^{\alpha}V(t)\le-\theta\|y(t)\|^2, \forall t\ge 0.$ Therefore, the conditions (ii) and (iii) in Lemma 1 are satisfied. Therefore, system (1) is fractional exponentially stable by Lemma 1.

Remark 1 Noted here that almost all of the existing results on exponential stability problems of dynamic systems with convex polytopic uncertainties are focused on integer-order systems [15]-[18], and few works are considered fractional-order systems subject to Caputo fractional derivative [19]-[21], not deal with fractional-order systems with conformable derivative. Theorem 1 has solved the problem for Hopfield FONNs subject to conformable fractional derivative and convex polytopic uncertainties for the first time.

When $N=1,$ we have the following systems

$$\begin{cases} T^{\alpha}y(t)=-\mathbf{A}y(t)+\mathbf{W}g(y(t)), t\ge 0 \\ y(0)=y_0. \end{cases} \tag{8}$$

According to Theorem 1, the following result is obtained.

Corollary 1 *The system* (8) *is fractional exponentially stable if there exist* $\mathbf{S}\in \mathrm{S}^{+}, \mathbf{P}\in \mathrm{S}^{++},$ *and a scalar* $\varepsilon>0$ *such that the following LMIs hold*

$$\begin{bmatrix} -\mathbf{A}^T\mathbf{P}-\mathbf{P}\mathbf{A}^T+\varepsilon\mathbf{L}^T\mathbf{L}+\mathbf{S} & \mathbf{P}\mathbf{W} \\ \mathbf{W}^T\mathbf{P} & -\varepsilon\mathbf{I} \end{bmatrix}<0.$$

Remark 2 The authors in [10] derived a stability condition in terms of matrix elements for system (8). In this paper, the stability condition in Corollary 1 is established in the form of LMIs.

We give a numerical example to show the less conservatism of our results.

Example 1 Consider the following Hopfield conformable FONNs with ring structure [22].

$$\begin{cases} T^{\alpha} y(t) = -\mathbf{A}y(t) + \mathbf{W}g(y(t)), t \geq 0 \\ y(0) = y_0, \end{cases} \quad (9)$$

where $\alpha \in (0,1]$, $y(t) = (y_1(t), y_2(t), y_3(t)) \in \mathbb{R}^3$, and

$$\mathbf{A} = diag\{a_1, a_2, a_3\} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \mathbf{W} = [w_{ij}]_{3\times 3} = \begin{bmatrix} 3 & 1 & -2.5 \\ -1 & 1.5 & 2 \\ -2.5 & 2 & -1 \end{bmatrix}.$$

We choose the activation function as follows

$$g(y(t)) = (\tanh(y_1(t)), \tanh(y_2(t)), \tanh(y_3(t)))^T \in \mathbb{R}^3.$$

Noted that the function $g(y(t))$ satisfies the condition (2) with $\mathbf{L} = diag\{1,1,1\}$. With the help of LMI Control Toolbox in MATLAB [15], we can find a solution of the condition in Corollary 1 as follows $\varepsilon = 378.8181$, and

$$P = \begin{bmatrix} 90.0484 & 14.2159 & 4.0217 \\ 14.2159 & 118.2542 & -7.6620 \\ 4.0217 & -7.6620 & 96.9751 \end{bmatrix}, S = \begin{bmatrix} 114.2390 & 114.4621 & 33.6395 \\ 114.4621 & 188.1833 & -63.1433 \\ 33.6395 & -63.1433 & 174.1761 \end{bmatrix}.$$

Therefor, system (8) is fractional exponentially stable for all $\alpha \in (0,1]$ by Corollary 1. However, the result in [10] cannot be handed in Example 1. Using some simple computation, we obtain $a_1 = 5, \sum_{l=1}^{3} \kappa_l |w_{1l}| = 6.5, a_2 = 4, \sum_{l=1}^{3} \kappa_l |w_{2l}| = 4.5, a_3 = 5, \sum_{l=1}^{3} \kappa_l |w_{3l}| = 5.5.$ So $\sum_{l=1}^{3} \kappa_l |w_{il}| > a_i (i = 1,2,3)$ fails to satisfy the condition $\sum_{l=1}^{3} \kappa_l |w_{il}| < a_i (i = 1,2,3)$ of Theorem 2 in [10].

4. Conclusion

We have solved fractional exponential stability problem for Hopfield neural networks subject to conformable derivative and convex polytopic uncertainties in this paper. By using the fractional Lyapunov theorem combined with LMIs techniques, a new sufficient condition for exponential stability has been derived. An example was given to show that our results are less conservative than those in the existing work. In the future works, we will investigate stability analysis of delayed neural networks with conformable fractional derivative.

REFERENCES

[1] V. T. Mai, C. H. Dinh, and T. H. Duong, "New results on robust finite-time passivity for fractional-order neural networks with uncertainties," *Neural Processing Letters*, vol. 50, no. 2, pp. 1065-1078, 2019.

[2] C. Z. Aguilar, J. F. Gómez-Aguilar, V. M. Alvarado-Martínez, and H. M. Romero-Ugalde, "Fractional order neural networks for system identification," *Chaos, Solitons & Fractals*, vol. 130, p.109444, 2020.

[3] B. Hu, Q. Song, and Z. Zhao, "Robust state estimation for fractional-order complex-valued delayed neural networks with interval parameter uncertainties: LMI approach," *Applied Mathematics and Computation*, vol. 373, p. 125033, 2020.

[4] K. Udhayakumar, F. A. Rihan, R. Rakkiyappan, and J. Cao, "Fractional-order discontinuous systems with indefinite LKFs: An application to fractional-order neural networks with time delays," *Neural Networks*, vol.145, pp. 319-330, 2022.
[5] S. Zhang, Y. Yu, and J. Yu, "LMI conditions for global stability of fractional-order neural networks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 10, pp. 2423-2433, 2017.
[6] Y. Yang, Y. He, Y. Wang, and M. Wu, "Stability analysis of fractional-order neural networks: an LMI approach," *Neurocomputing*, vol. 285, pp. 82-93, 2018.
[7] H. Zhang, R. Ye, S. Liu, J. Cao, A. Alsaedi, and X. Li, "LMI-based approach to stability analysis for fractional-order neural networks with discrete and distributed delays," *International Journal of Systems Science*, vol. 49, no. 3, pp. 537-545, 2018.
[8] W. Huang, Q. Song, Z. Zhao, Y. Liu, and F. E. Alsaadi, "Robust stability for a class of fractional-order complex-valued projective neural networks with neutral-type delays and uncertain parameters," *Neurocomputing*, vol. 450, pp. 399-410, 2021.
[9] M. Syed Ali, M. Hymavathi, S. Saroha, and R. Krishna Moorthy, "Global asymptotic stability of neutral type fractional-order memristor-based neural networks with leakage term, discrete and distributed delays," *Mathematical Methods in the Applied Sciences*, vol. 44, no. 7, pp. 5953-5973, 2021.
[10] A. Kütahyalıoglu and F. Karakoç, "Exponential stability of Hopfield neural networks with conformable fractional derivative," *Neurocomputing*, vol. 456, pp. 263-267, 2021.
[11] T. T. H. Nguyen, H. S. Nguyen, and V. T. Mai, "LMI conditions for fractional exponential stability and passivity analysis of uncertain Hopfield conformable fractional-order neural networks," *Neural Processing Letters*, vol. 54, no. 2, pp. 1333-1350, 2022.
[12] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, "A new definition of fractional derivative," *Journal of Computational and Applied Mathematics*, vol. 264, pp. 65-70, 2014.
[13] A. Souahi, A. B. Makhlouf, and M. A. Hammami, "Stability analysis of conformable fractional-order nonlinear systems," *Indagationes Mathematicae*, vol. 28, no. 6, pp. 1265-1274, 2017.
[14] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, Linear matrix inequalities in system and control theory, *Society for Industrial and Applied Mathematics*, 1994.
[15] Y. He, M. Wu, J. H. She and G. P. Liu, "Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties," *IEEE Transactions on Automatic Control*, vol. 49, no. 5, pp. 828-832, 2004.
[16] Y. He, Q. G. Wang, and W. X. Zheng, "Global robust stability for delayed neural networks with polytopic type uncertainties," *Chaos, Solitons and Fractals*, vol. 26, pp. 1349-1354, 2005.
[17] P. Balasubramaniam and S. Lakshmanan, "Delay-interval-dependent robust-stability criteria for neutral stochastic neural networks with polytopic and linear fractional uncertainties," *International Journal of Computer Mathematics*, vol. 88, no. 10, pp. 2001-2015, 2011.
[18] K. Moezzi and A. G. Aghdam, "Delay-dependent robust stability analysis for switched time-delay systems with polytopic uncertainties," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 11, pp. 1623-1637, 2015.
[19] S. Adelipour, A. Abooee, and M. Haeri, "LMI-based sufficient conditions for robust stability and stabilization of LTI-fractional-order systems subjected to interval and polytopic uncertainties," *Transactions of the Institute of Measurement and Control*, vol. 37, no. 10, pp. 1207-1216, 2015.
[20] C. H. Dinh, V. T. Mai, and T. H. Duong, "New results on stability and stabilization of delayed Caputo fractional-order systems with convex polytopic uncertainties," *Journal of Systems Science and Complexity*, vol. 33, no. 3, pp. 563-583, 2020.
[21] R. Abolpour, M. Dehghani, and M. S. Tavazoei, "Reducing conservatism in robust stability analysis of fractional-order-polytopic systems," *ISA transactions*, vol. 119, pp. 106-117, 2022.
[22] E. Kaslik and S. Sivasundaram, "Nonlinear dynamics and chaos in fractional-order neural networks," *Neural Networks*, vol. 32, pp. 245-256, 2012.