DYNAMICAL ANALYSIS OF A PREDATOR - PREY MODEL USING HUNTING STRATEGIES

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ABSTRACT

In this paper, we establish a new prey-predator model using game theory with solitary hunting or pack hunting strategies. The model includes a fast-time scale and a slow-time scale to investigate the effect of predator behavior on the ecosystem. In our model, we assume that the switch between hunting strategies and hawk-dove tactics happens on a fast-time scale, while the development of the species of prey intrinsic growth, predator mortality, and hunting process, takes place on a slow-time scale. We use the differential equations theory and the aggregated method to study the model's well-posedness and the properties of its solution, such as positivity, boundedness, and stability. It is shown that the coexistence of prey and predator might be in a steady state or a chaotic state. Some numerical simulations illustrate the theoretical results in cases of stable equilibrium and chaotic equilibrium are given. Discussions about predators' behavior and the ecosystem's development are also provided.

PHÂN TÍCH HỆ ĐỘNG LỰC THÚ MỒI SỬ DỤNG CHIẾN THUẬT SĂN MỒI

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THÔNG TIN BÀI BÁO

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TỪ KHÓA

Mô hình thú mồi
Phương pháp tổ hợp biến
Lý thuyết trò chơi
Chiến thuật săn mồi
Phân tích ổn định

TÓM TẮT

Trong bài báo này, chúng tôi xây dựng một mô hình thú mồi mới với tập tính săn mồi theo bầy đàn hoặc đơn lẻ sử dụng lý thuyết trò chơi. Mô hình bao gồm hai thang thời gian nhanh và chậm nhằm khảo sát ảnh hưởng của hành vi săn mồi đối với hệ sinh thái. Giả sử rằng, sự chuyển đổi chiến thuật săn mồi diễn ra trên thang thời gian nhanh và sự phát triển loài như quá trình tăng trưởng nội tại của loài mồi, quá trình chết tự nhiên của loài thú và quá trình săn bắt mồi được xét trên thang thời gian chậm. Lý thuyết phương trình vi phân và phương pháp tổ hợp biến được sử dụng để khảo sát tính đặt chỉnh và một số tính chất định tính của nghiệm bài toán như tính chất dương, tính bị chặn, tính chất ổn định. Các phân tích hệ động lực chỉ ra rằng, sự sinh tồn đồng thời của hai loài có thể đạt trạng thái ổn định hoặc trạng thái hỗn loạn. Chúng tôi mô phỏng số cho mô hình, minh hoạ trường hợp điểm cân bằng ổn định và điểm cân bằng hỗn loạn. Trên cơ sở đó, một số bình luận về hành vi của loài thú và sự phát triển của hệ sinh thái đã được đưa ra.

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1. Introduction

Biological population dynamics is very attractive to most of mathematical biologists. There are different ways to explore the population dynamics. Evolutionary game theory is one of the most useful ways in studying the dynamics involving the behavior of the objects. In 1973, an application of game theory in exploring the biological evolution is first introduced by John Maynard Smith and George Robert Price [1]. John Maynard Smith introduced his work on theory of evolutionary stable strategy (ESS) in his book ”Evolution and the Theory of Games” [2]. ESS becomes familiar to scientists [3],[4].

The combination of predator - prey model and game theory is effectively used to explore the effect of predator behavior on the dynamics. Since 1998, Auger et al. have presented some results on the predator-prey models with the hawk - dove game (aka the chicken game) [5]–[7]. A model with N-person hawk - dove games which considered the fighting between N hawks for food was studied in [8] by Wei Chen et al. Recently, some results on predator - prey model using Rock-Paper-Scissor strategies have been established [9]–[11].

In this paper, we study a predator-prey model combining the classical model in slow time scale as in [6] with the changing behavior of predator following the chicken game in fast time scale. We consider a predator and prey model in which the predator uses hawk-dove tactics in combination with solitary hunting or pack hunting behavior. There is a lack of research on this problem till now.

The model in this paper includes population of prey, population of predator, behavior of predator in catching prey: in group or alone, aggressive or not. In fact, when predators see the same prey at the same time, they might cooperate with the others to fight for food or retreat to avoid fighting. Which tactics in which conditions would be more beneficial? That is the reason why we are interested in exploring the model with the different behaviors of the predator.

The rest of this paper is organized as follows: In Section 2, the mathematical model with ecological assumptions and the meaning of parameters are described. Section 3 derives the positivity and the boundedness of solutions and presents the aggregation model. In Section 4, the stability analysis of the equilibrium points is presented. Section 5 is devoted to the illustration of some numerical simulations of our theoretical results. Finally, we give some discussions and the biological significance of our analytical findings in Section 6.

2. Model formulation

Let t and τ be the notations of time on slow and fast time scale, respectively. We denote $n(t)$ and $p(t)$ as the densities of the prey and predator, respectively, at time t. We shall build up a model describing what happens on both time scales: the changes of predation strategies on the fast time scale, and the hunting process and development of the species on the slow one.

2.1. Hunting game dynamics on the fast time scale

At first, we will introduce the rules of the hunting game. Assuming that at time t, predators are divided into two sub-groups, named the pack predators and the solitary predators. The pack group includes all predators that choose to cooperate with the others to fight for food. The solitary group includes all predators that choose to independently fight another single or to retreat to avoid combat. Let denote $p_A(t)$ and $p_L(t)$ as the pack predator and solitary predator densities, respectively, at time t. The total density of predators is given by

$$p(t) = p_A(t) + p_L(t). \tag{1}$$

On the fast time scale, predators fight for a captured prey. During an encounter, one individual must choose either to cooperate with the others in a herd or to hunt independently. Furthermore, we assume that herds have the same size and let $q \geq 2$ is the number of the individuals of one. The game describes this process of conflicts between two predation herds and between two single predators. The gain G of the game corresponds to the prey amount that the predators dispute over during each unit of time. In this model, we assume that the amount of prey killed per unit of time per one predator is proportional to the density of prey with a proportional coefficient $a > 0$. In other words, the gain $G(n)$ of the game is the amount of prey killed which is defined as follow:

$$G(n) = an. \tag{2}$$

Let $C > 0$ be the cost due to fighting between herds and between individuals.

Now, we denote A, L for pack group and solitary group, respectively; a coefficient M_{lk} of the payoff matrix corresponds to the gain that is obtained by an individual playing tactic l against an individual playing tactic k; $l,k \in \{A,\ L\}$. We assume that the average gain and the average cost due to injuries are equally divided by the individuals that have the same tactic, $M_{AA} = \dfrac{G-C}{2q}$ and $M_{LL} = \dfrac{G-C}{2}$. When one individual meets a herd, it always retreats and lets the group obtains the gain, which means $M_{LA} = 0$, $M_{AL} = \frac{G}{q}$. Therefore, the payoff matrix M of the game is the following one:

$$M = \begin{pmatrix} \frac{G-C}{2q} & \frac{G}{q} \\ 0 & \frac{G-C}{2} \end{pmatrix}. \tag{3}$$

Let x and y be the proportion of pack predators and solitary ones in the total predators.

$$x = \frac{p_A}{p}, \; y = \frac{p_L}{p} = 1 - x. \tag{4}$$

Thus, at time t, the gain Δ_A of an individual that always choose to cooperate with the others and the gain Δ_L of one always choose to be single are given:

$$\Delta_A = \begin{pmatrix} 1 & 0 \end{pmatrix} M \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \Delta_L = \begin{pmatrix} 0 & 1 \end{pmatrix} M \begin{pmatrix} x \\ y \end{pmatrix}. \tag{5}$$

Therefore, the average gain of an individual playing the two tactics in proportions x and y is the following one:

$$\Delta = \begin{pmatrix} x & y \end{pmatrix} M \begin{pmatrix} x \\ y \end{pmatrix}. \tag{6}$$

Naturally, a predator individual would choose a tactic that helps it get more benefit. It means that if the gain for one tactic is greater (or smaller) than the average gain, the size of that tactic group should be increased (or decreased). In additional, we assume that the game is fast in comparison to other processes. With these assumptions, the equations for the tactic groups are given:

$$\frac{dp_k}{d\tau} = p_k \left(\Delta_k - \Delta\right), \quad k \in \{A, \, L\}. \tag{7}$$

In the next part, we shall consider processes happen on slow time scale such as the predator mortality, prey growth and process of prey capturing.

2.2. Dynamics of prey density on the slow time scale

In the model, we assume that the intrinsic growth of prey population follows logistic function with a natural growth rate r and an environmental carrying capacity constant K. The density of preys also depends on the number of preys killed by predators, which is proportional to the size of prey population and predator population. Specifically, we use a Lotka - Volterra functional response type I with the intake rate a mentioned before in the gain G of game. Thus, the equation for the dynamics of prey is given as follows:

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{K}\right) - anp, \tag{8}$$

where t corresponds to the slow time scale. The relationship between the two time - scales is $t = \varepsilon\tau$.

2.3. Dynamics of predator densities on the slow time scale

For predators, we assume that the growth depends on only the number of preys killed. That means predators grow by preys they catch and naturally die with mortality rate $\mu > 0$. We also assume that each tactic group as a proportion of predators is governed by the same rules. That leads to the following equation:

$$\frac{dp_k}{dt} = -\mu p_k + \alpha \Delta_k p_k, \quad k \in \{A, \, L\}. \tag{9}$$

in which $\alpha > 0$ is a conversion constant of gain into biomass of predators. In other words, the growth rate of each subgroup is proportional to the average payoff obtained by an individual in that subgroup. A pack predator individual can encounter either another one in different herd in proportion x and gets the gain $\dfrac{G-C}{2q}$, or a single individual in proportion y, gets $\dfrac{G}{q}$. Consequently, the growth of the pack predator sub-population obeys the following equation:

$$\frac{dp_A}{dt} = -\mu p_A + \alpha \begin{pmatrix} 1 & 0 \end{pmatrix} M \begin{pmatrix} x \\ y \end{pmatrix} p_A = -\mu p_A + \alpha \left(\frac{G-C}{2q} \cdot \frac{p_A}{p} + \frac{G}{q} \cdot \frac{p_L}{p} \right) p_A. \tag{10}$$

Similarly, we obtain an equation that rules the population of the solitary predator subgroup with notice that a single predator will retreat when it meets a group, gets no gain, and will equally share the gain and cost when it meets other solitary individual.

$$\frac{dp_L}{dt} = -\mu p_L + \alpha \begin{pmatrix} 0 & 1 \end{pmatrix} M \begin{pmatrix} x \\ y \end{pmatrix} p_L = -\mu p_L + \alpha \left(\frac{G-C}{2} \cdot \frac{p_L}{p} \right) p_L. \tag{11}$$

The predator population and prey population growths are assumed to be on slow time scale. This is matched with the fact that the fighting between predators frequently happens while the number of preys captured each day is much smaller than the population.

2.4. The complete slow-fast model

The complete model that combines all processes in slow and fast time scale reads as:

$$\begin{cases} \varepsilon \dfrac{dn}{dt} = \varepsilon \left[rn \left(1 - \dfrac{n}{K} \right) - an(p_A + p_L) \right] \\ \varepsilon \dfrac{dp_k}{dt} = p_k(\Delta_k - \Delta) + \varepsilon \left[-\mu p_k + \alpha \Delta_k p_k \right], \quad k \in \{A, L\} \end{cases} \tag{12}$$

where $\varepsilon \ll 1$ is a small parameter. We also use the fast time scale τ to rewrite the complete model (12):

$$\begin{cases} \dfrac{dn}{d\tau} = \varepsilon \left[rn \left(1 - \dfrac{n}{K} \right) - an(p_A + p_L) \right] \\ \dfrac{dp_A}{d\tau} = p_A(\Delta_A - \Delta) + \varepsilon \left[-\mu p_A + \alpha \left(\dfrac{G-C}{2q} \cdot \dfrac{p_A}{p} + \dfrac{G}{q} \cdot \dfrac{p_L}{p} \right) p_A \right] \\ \dfrac{dp_L}{d\tau} = p_L(\Delta_L - \Delta) + \varepsilon \left[-\mu p_L + \alpha \left(\dfrac{G-C}{2} \cdot \dfrac{p_L}{p} \right) p_L \right] \end{cases} \tag{13}$$

The model (13) clearly shows that the population of prey as well as predator population change very little in fast time scale when ε is small enough. The game dynamics on fast time scale shows the changes in the sizes of the two tactical groups. This model is a three-dimensional system of ordinary differential equations.

3. Positivity, boundedness and aggregated model

3.1. Positivity and boundedness

In this part, we get some properties for the solutions of the complete model system (13) which relate to the positivity and boundedness.

Theorem 0.1. *All the solutions of the complete model (13), which start in $\mathbb{R}^3_+$ are always positive and bounded.*

Theorem 0.2. *For any given initial value in $\mathbb{R}^3_+$, the complete model (13) has a unique positive solution.*

These properties guarantee the meaning of exploring the model because of the positivity and boundedness of populations of prey and predators.

3.2. The aggregated model

We shall use the aggregation method, referred to [5],[12],[13], to reduce the dimension of the complete system (13) of the three equations into the aggregated model of the two equations.

3.2.1. Fast equilibrium

The first step of the method is to neglect the small terms $O(\varepsilon)$ and to look for the existence of a stable equilibrium of the game dynamics which happen on fast time scale.

$$\frac{dp_k}{d\tau} = px\left(\Delta_k - \Delta\right),\ k \in \{A,\ L\} \tag{14}$$

On fast time scale, the densities of the prey and the predator, n and $p = p_A + p_L$ respectively, can be considered as constants. Moreover, because $x,\ y$ are proportion of pack predator subgroup and solitary predator subgroup in total predators, which means $x + y = 1$, we can reduce the system (14) of the two dimension to one equation that rules the pack predator group. Hence, the game dynamics is ruled by the following equation:

$$\frac{dx}{dt} = x(1-x)\left[\left(\frac{G-C}{2q} - \frac{G}{q} + \frac{G-C}{2}\right)x + \frac{G}{q} - \frac{G-C}{2}\right]. \tag{15}$$

The equation (15) has three equilibria of 0, 1, and $x^* = \dfrac{(q-2)G - qC}{(q-1)G-(q+1)C}$. We now consider the cases as follows:

Case A. $q > 2 \Rightarrow C < \dfrac{q+1}{q-1}C < \dfrac{qC}{q-2}$.

According to parameters values, three cases can occur:

A1. $G > \dfrac{qC}{q-2} \Rightarrow 0 < x^* < 1$. $0, 1$ are stable; and x^* is unstable.

A2. $C < G < \dfrac{qC}{q-2} \Rightarrow x^* \notin [0;1]$. 1 is stable; 0 is unstable.

A3. $G < C \Rightarrow x^* \in [0,1]$. x^* is stable; $0, 1$ are unstable.

Case B. $q = 2 \Rightarrow x^* = \dfrac{-2C}{G-3C}$. We have three cases as follows:

B1. $G > 3C \Rightarrow x^* < 0 \Rightarrow x^* \notin [0;1]$. 1 is stable; 0 is unstable.

B2. $C < G < 3C \Rightarrow x^* > 1 \Rightarrow x^* \notin [0;1]$. 1 is stable; 0 is unstable.

B3. $G < C \Rightarrow 0 < x^* < 1$. x^* is stable; 0;1 are unstable.

3.2.2 Aggregated model

The second step of the aggregation method is to substitute the fast equilibrium and add the two predator equations in the complete model with the assumption that the fast process is at the fast equilibrium. Thus, with the notations $\bar{x}$ for the stable equilibrium, $\bar{p}_k, \bar{\Delta}_k, k \in \{A,\ L\}$ respectively stand for $p_k, \Delta_k, k \in \{A,\ L\}$ at the stable equilibrium, we have the aggregated model

$$\begin{cases} \dfrac{dn}{dt} = n\left[r\left(1 - \dfrac{n}{K}\right) - ap\right] \\[2ex] \dfrac{dp}{dt} = \left[-\mu + \alpha\left(\bar{\Delta}_A\bar{x} + \bar{\Delta}_L(1-\bar{x})\right)\right]p. \end{cases} \tag{16}$$

We have three fast equilibria and the gain depends on the prey density $G = an$. Therefore, we obtain three aggregated models which are valid on three different domains of phase plane.

- **Case A**, $q > 2$.
  Model I: $n > \dfrac{q}{q-2}\dfrac{C}{a}$, $\bar{x} = 0$ is stable in Eq. (15),

$$\begin{cases} \dfrac{dn}{dt} = n\left[r\left(1 - \dfrac{n}{K}\right) - ap\right] \\[2ex] \dfrac{dp}{dt} = \left(-\mu + \alpha\dfrac{an-C}{2}\right)p. \end{cases} \tag{17}$$

Model II: $n > \frac{C}{a}$, $\quad \bar{x} = 1$ is stable in Eq. (15),

$$\begin{cases} \dfrac{dn}{dt} = n\left[r\left(1 - \dfrac{n}{K}\right) - ap\right] \\ \dfrac{dp}{dt} = \left(-\mu + \alpha\dfrac{G-C}{2q}\right)p \end{cases} \tag{18}$$

Model III: $n < \frac{C}{a}$, $\quad \bar{x} = x^*$ is stable in Eq. (15),

$$\begin{cases} \dfrac{dn}{dt} = n\left[r\left(1 - \dfrac{n}{K}\right) - ap\right] \\ \dfrac{dp}{dt} = \dfrac{\alpha p}{2}.\dfrac{1}{(q-1)an - (q+1)C}.H(n) \end{cases} \tag{19}$$

in which $H(n) = a^2n^2 - \left(\frac{2\mu(q-1)}{\alpha} + 2C\right)an + \frac{2\mu(q+1)C}{\alpha} + C^2$.

When $n > \frac{qC}{(q-2)a}$, we have two models, namely systems (17) and (18). If $x < x^*$, the model I, Eq. (17) is governing and if $x > x^*$, the model II, Eq. (18) is ruling.

- **Case B**, $q = 2$.

  When $n > \frac{C}{a}, \bar{x} = 1$ is stable in Eq. (15), model II, Eq. (18) is governing. When $n < \frac{C}{a}, \bar{x} = x^*$ is stable in Eq. (15), model III, Eq. (19) is ruling.

In general, there are two cases as follows:

- Case 1: Model III is valid on $\left\{n, n < \frac{C}{a}\right\}$. Model II is valid on $\left\{n, n > \frac{C}{a}\right\}$.
- Case 2: Model III is valid on $\left\{n, n < \frac{C}{a}\right\}$. Model II is valid on $\left\{n, \frac{C}{a} < n < \frac{qC}{(q-2)a}\right\}$. And Model I is valid on $\left\{n, n > \frac{qC}{(q-2)a}\right\}$.

In the phase space (n, p), besides the vertical line $n = \frac{C}{a}$ at which these models connect in both cases, in case 2, the other connected line is $n = \frac{qC}{(q-2)a}$.

4. Stability analysis of the aggregated model

The first equation of the three models are the same. That leads to the same $n-$ nullclines: $n = 0$ and $p = \frac{r}{a}\left(1 - \frac{n}{K}\right)$. The $p-$ nullclines which depend on parameters values can be found from the equations $\frac{dp}{dt} = 0$ of the three models. For model I, system (17), we have two nullclines, $p = 0$ and the vertical line $n = n_1^* = \frac{2\mu + \alpha C}{\alpha a}$. For model II, Eq. (18), there are two $\frac{dp}{dt} = 0$ nullclines, $p = 0$ and the vertical line $n = n_2^* = \frac{2\mu q + \alpha C}{\alpha a}$. For model III, system (19), if $\mu(q-1)^2 - 4\alpha C < 0$ then $p = 0$ is the only $p-$ nullcline and if $\mu(q-1)^2 - 4\alpha C > 0$ then there are three nullclines $p = 0$; $n = n_3^*$; $n = n_4^*$, where

$$n = n_{3;4}^* = \frac{\alpha C + \mu(q-1) \pm \sqrt{\mu^2(q-1)^2 - 4\alpha\mu C}}{\alpha a}.$$

Those vertical nullclines are ordered as follows: $\frac{C}{a} < n_3^* < n_4^* < n_2^*$, and $\frac{C}{a} < n_1^* < n_2^*$. Thus, n_2^* is always in the domain of model II, Eq. (18) while n_3^*, n_4^* are always not in the domain of model III, Eq. (19). In case 1, $q > 2$, n_1^* is in the domain of model I, system (17), if $n_1^* > \frac{qC}{q-2}$. There are up to six equilibria. Two of them, $(0,0);(K;0)$ always exist and four others (n_i^*, p_i^*), $i \in \{1;2;3;4\}$ where $p_i^* = \frac{r}{a}\left(1 - \frac{n_i^*}{K}\right)$ can be found in positive quadrant if $n_i^* < K$. In any cases, $(0,0)$ is a saddle point.

According to the position of K with respect to the other equilibria as well as the connected lines, in each case, it is divided into some sub-cases.

Case 1: Fig. 1(a) corresponds to the sub-case $K < \frac{C}{a}$, the equilibrium $(K,0)$ is asymptotically stable which mean that the predator gets extinct, the prey tends to its carrying capacity. Fig. 1(b) corresponds to the sub-case $\frac{C}{a} < K < n_2^*$, the equilibrium $(K,0)$ is a stable node, the prey tends to its carrying capacity, the predator dies out. Fig. 1(c) corresponds to the sub-case $\frac{C}{a} < n_2^* < K$, the equilibrium $(K,0)$ is a saddle and the equilibrium (n_2^*, p_2^*) is a sink, thus the predator and the prey coexist.

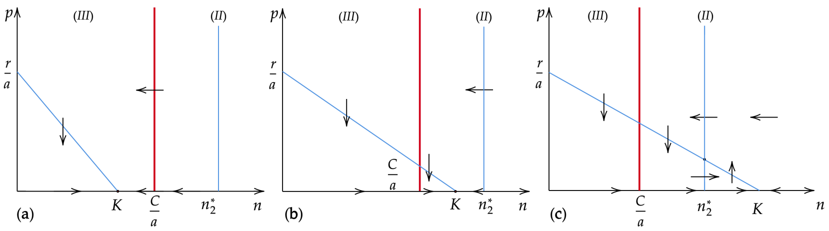

Figure 1. *Phase portrait of aggregated system in case 1: (a)* $K < \frac{C}{a}$*, (b)* $\frac{C}{a} < K < n_2^*$*, (c)* $K > n_2^*$*.*

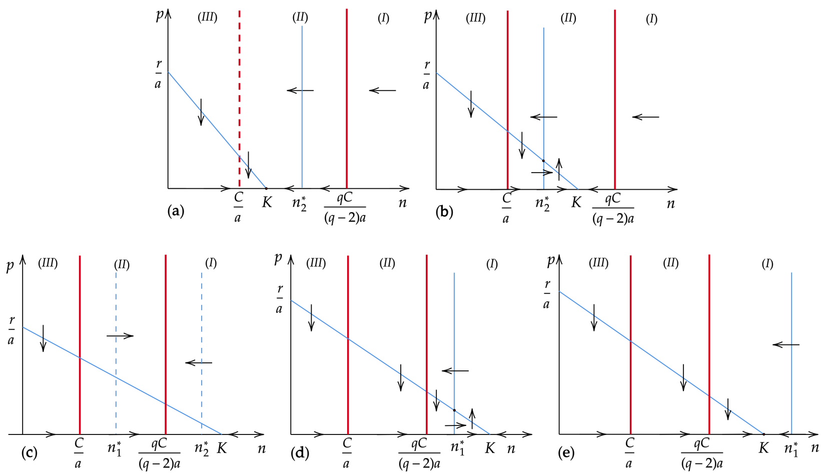

Figure 2. *Phase portrait of aggregated system in case 2 (a)* $K < min\left\{n_2^*, \frac{qC}{(q-2)a}\right\}$, *(b)* $n_2^* < K < \frac{qC}{(q-2)a}$, *(c)* $n_1^* < \frac{qC}{(q-2)a} < min\{K, n_2^*\}$, *(d)* $\frac{qC}{(q-2)a} < n_1^* < K$, *(e)* $\frac{qC}{(q-2)a} < K < n_1^*$.

Case 2: Fig 2.(a) corresponds to the sub-case $K < \min\left\{\frac{qC}{(q-2)a}, n_2^*\right\}$, the equilibrium $(K,0)$ is a stable point which means that the predator becomes extinct, the prey approaches its carrying capacity. Fig 2.(b) corresponds to the sub-case $\frac{C}{a} < n_2^* < K < \frac{qC}{(q-2)a}$, the equilibrium $(K,0)$ is a saddle and the equilibrium (n_2^*, p_2^*) is a stable focus, so the prey and the predator coexist. Fig 2.(c) corresponds to the sub-case $n_1^* < \frac{qC}{(q-2)a} < K$, the equilibrium $(K,0)$ is a saddle. There is no stable point. The system has a chaotic behavior solution. The density of prey is pushed back and forth through the connected line between the domains of the model II and I. Fig 2.(d) corresponds to the sub-case $\frac{qC}{(q-2)a} < n_1^* < K$. The equilibrium $(K,0)$ is a saddle. The equilibrium (n_1^*, p_1^*) is asymptotically stable. Therefore, the prey and the predator are in coexistence. Fig 2.(e) corresponds to the sub-case $\frac{qC}{(q-2)a} < K < n_1^*$. The equilibrium $(K,0)$ is a sink. Therefore, the prey approaches its carrying capacity and the predator goes extinct.

5. Numerical simulations

In this section, we preview numerical simulations to illustrate the theoretical results in previous sections. The first two figures 3 and 4 in this section show the behavior of the solution, specifically, prey density and predator density, of both complete model and aggregated model in the same initial conditions and the same parameter values. As ε changes and gets a small value, it can be found the similarity in the value of the solution while the time scale goes on the infinitive. From now, we use the aggregated model to study the behavior of the complete model. The next four figures 5, 6, 7 and 8 illustrate the cases that prey, and predator coexist when the max gain greater than the cost of competing. Figure 5, 6 show the behavior of the densities of prey and predator along timeline while the figure 7 and 8 show the phase portrait of the systems. It has been seen that the densities might start at different points but end up at the same one. The last three figures 10, 11 and 12 represent the case when chaotic phenomenon happens. In this case the predator and prey coexist in unstable state. The densities pushed back and forth between the two domains of the systems II and I.

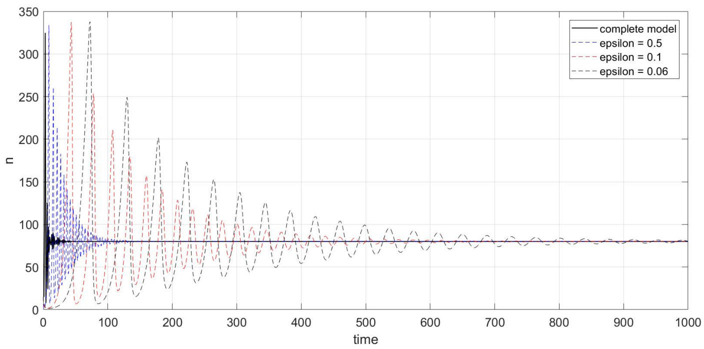

Figure 3. *Prey density of complete model and aggregated model with different value of ε*

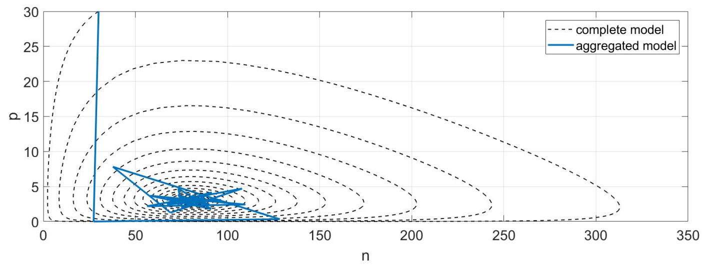

Figure 4. *Behavior of solutions of the complete model and aggregated model with $\varepsilon = 0.1$*

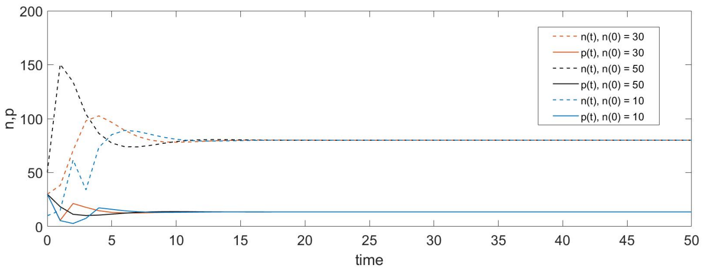

Figure 5. *Coexistence at different initial density of prey in case* $aK > C$.

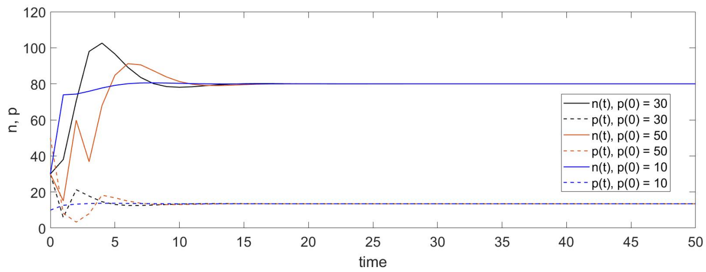

Figure 6. *Coexistence at different initial density of predators in case* $aK > C$.

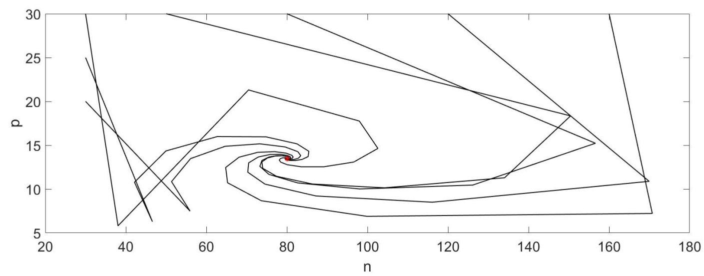

Figure 7. *Phase portrait - Coexistence at different initial conditions in case* $aK > C$.

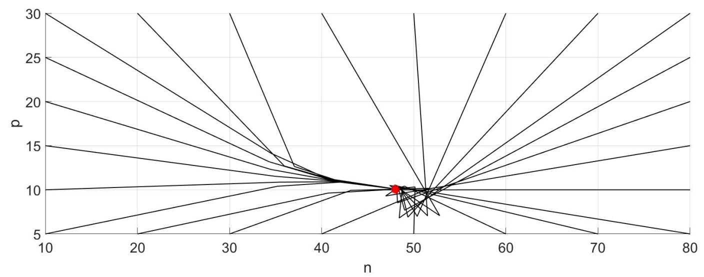

Figure 8. *Phase portrait. Coexistence at different initial conditions in case* $aK > qC/(q-2)$.

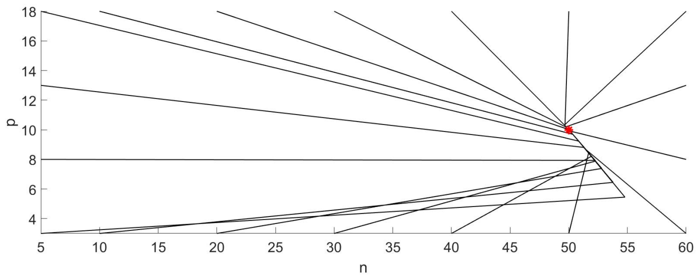

Figure 9. *Phase portrait. Coexistence at different initial conditions in case* $aK < qC/(q-2)$.

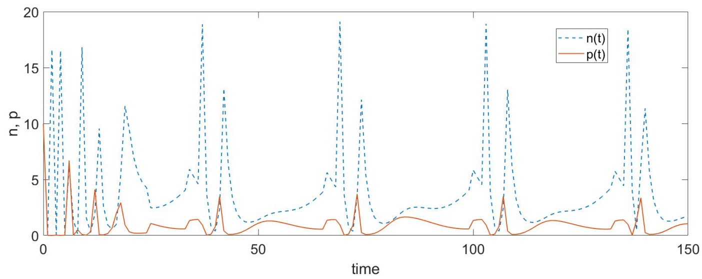

Figure 10. *Coexistence in chaotic state in a short period of time*

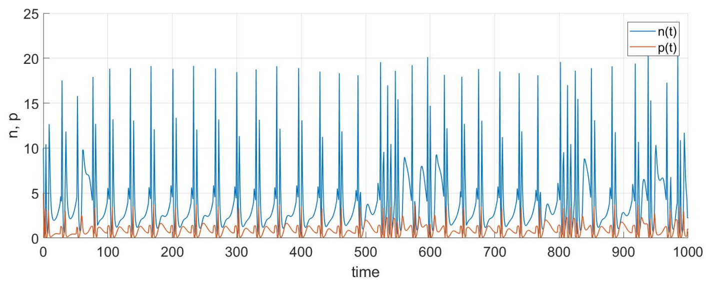

Figure 11. *Coexistence in chaotic state in a long period of time*

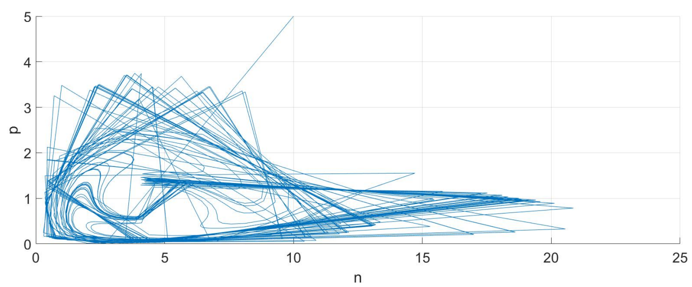

Figure 12. *Behaviors of the dynamics system in chaotic state*

6. Discussion and Conclusion

In this paper, we have established a new prey-predator model with hunting strategies using modified hawk-dove tactics on two-time scales. From the results of the stability analysis of the model, it can be concluded that if the gain is less than the cost of the competition, predators might switch hunting tactics. Otherwise, if the gain is greater than the cost of fighting, predators tend to choose the same tactic, i.e., all individuals choose the herd strategy, or all individuals choose the solitary strategy. From the simulations and stability analysis, it can be seen that When the maximum gain of one sub-group is much smaller than the total cost, the predator becomes extinct, and prey reaches the environment capacity no matter which strategies are chosen. When the maximum gain of one sub-group is greater than the total cost, the prey and the predator coexist either in a steady state or chaotic state. In a steady state, the sizes of the prey population and predator population do not change, while densities are pushed back and forth in a chaotic state. Some issues can be further explored, such as the properties of the chaotic state of the dynamical system, the effect of the size of the herd on the ecosystem, etc. We leave this part for future work.

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