RESEARCH AND APPLICATION OF BALANCED TRUNCATION ALGORITHM TO REDUCE ORDER FOR ROBUST CONTROLLER OF SYNCHRONOUS GENERATOR LOAD ANGLE

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ABSTRACT

Sustainable control plays an important role in ensuring stability and performance of the synchronous generator load angle. However, high-order controllers can lead to many limitations such as complex processing software, slow response, bulky and costly hardware implementation. To address these issues and achieve economic and technical objectives, it is necessary to reduce the order of the controller. The author's team applied the Balanced Truncation Algorithm (BlTA) to reduce the stable subsystem order of the sustainable controller for the synchronous generator load angle. Simulation results on Matlab show the effectiveness of BlTA in reducing the order of the initial control system (stable part) from 22 to orders 2, 3, and 4. Among them, the fourth-order system provides good time and frequency responses while maintaining a small error between the reduced and original systems. Therefore, a reduced-order system can be chosen to replace a high-order control system within an appropriate range, resulting in faster response time, simplifying implementation and programming while still meeting some allowed technical specifications.

NGHIÊN CỨU, ÁP DỤNG THUẬT TOÁN CẮT NGẮN CÂN BẰNG ĐỂ GIẢM BẬC CHO BỘ ĐIỀU KHIỂN BỀN VỮNG GÓC TẢI MÁY PHÁT ĐỒNG BỘ

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TỪ KHÓA

Điều khiển bền vững
Thuật toán cắt ngắn cân bằng
Hệ thống bậc cao
Giảm bậc mô hình
Góc tải máy phát đồng bộ

TÓM TẮT

Điều khiển bền vững đóng vai trò quan trọng để đảm bảo sự ổn định và hiệu suất của góc tải máy phát điện đồng bộ. Tuy nhiên, bậc cao của bộ điều khiển có thể dẫn đến nhiều hạn chế, như phần mềm xử lý phức tạp, đáp ứng chậm, phần cứng triển khai cồng kềnh và tốn kém. Nhằm giải quyết các vấn đề này, đảm bảo các mục tiêu kinh tế và kỹ thuật, ta cần giảm bậc cho bộ điều khiển. Nhóm tác giả đã áp dụng thuật toán cắt ngắn cân bằng Balanced Truncation Algorithm (BlTA), để giảm bậc phân hệ ổn định cho bộ điều khiển bền vững của góc tải máy phát điện đồng bộ. Kết quả mô phỏng trên Matlab cho thấy hiệu quả của BlTA trong việc giảm bậc hệ thống điều khiển (phần ổn định) ban đầu có bậc 22 xuống các bậc 2, bậc 3 và bậc 4. Trong số đó, hệ thống bậc 4 cho đáp ứng thời gian và tần số tốt, đồng thời sai số giữa hệ giảm bậc và hệ gốc nhỏ. Do đó có thể lựa chọn hệ giảm bậc để thay thế cho hệ thống điều khiển bậc cao trong phạm vi phù hợp, thời gian đáp ứng nhanh hơn, đơn giản hóa trong cài đặt, lập trình, trong khi vẫn đáp ứng một số chỉ tiêu kỹ thuật cho phép.

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1. Introduction

Robust control is a control theory that aims to achieve stable and predictable performance in uncertain and varying environments. It involves designing control systems that can handle changes in parameters, disturbances, and modeling errors. Robust control is used in many applications such as aerospace, chemical processes, robotics, and power systems. In the context of synchronous generator load angle control, robust control aims to ensure stable operation of the generator despite uncertainties in the system.

However, implementing robust control for synchronous generator load angle has some disadvantages due to its high order. The high order leads to difficulty in installation, slow response, and cumbersome and expensive implementation hardware. Therefore, reducing the model order is necessary to achieve economic and technical targets. Many methods of model order reduction are available, including the Balanced Truncation Algorithm (BlTA) [1]. This algorithm has several advantages such as being computationally efficient, providing accurate reduced-order models, and being easy to implement.

The BlTA is a model order reduction technique that balances the energy of the system before truncating the model. It involves computing the system's controllability and observability gramians, diagonalizing them, and then truncating the system's states with the least energy. The algorithm guarantees stability and accuracy in the reduced-order model while preserving the essential dynamics of the original system.

The BlTA has been extensively researched, developed, and applied to various systems, including mechanical, electrical, and aerospace systems, etc. Researchers continue to improve the algorithm by incorporating different techniques, and system identification, and different variants of the algorithm have been proposed to improve the performance of the reduced-order model, such as: The letter [2] proposes an adaptive truncation order selection method for calculating eigenvalues of linear time-periodic models for small-signal stability analysis of converter-dominated power systems. The proposed method aims to balance computational accuracy and efficiency. The effectiveness of the proposed methodology is validated through strict theoretical analysis and case studies on a-following voltage source converter and a modified IEEE 13-bus system. The article [3] proposes a method that uses balanced model reduction and greedy optimization to determine sensor and actuator locations that optimize observability and controllability. The method involves using greedy matrix QR pivoting on the dominant modes of the direct and adjoint balancing transformations to optimize scalar measures of observability and controllability. The method's runtime scales linearly with the state dimension, making it feasible for high-dimensional systems. The method's effectiveness is demonstrated using the linearized Ginzburg-Landau system, where it approximates known optimal placements computed using costly gradient descent methods. The article [4] provides a wide range of systems where balanced truncation guarantees a minimal internally positive system. The article [5] proposes a new algebraic observability Gramian based on the Hilbert space adjoint theory. The proposed Gramian satisfies a particular type of generalized Lyapunov equation and is connected to energy functions. This connection enables one to find states that are difficult to control and observe through a balancing transformation, which can be truncated to create reduced-order models. The article derives error bounds based on H_2 energy considerations, which depend on the neglected singular values. The BlTA is used to reduce the transfer function of the DFIG (Doubly-Fed Induction Generator) and obtain an analytical description of the oscillation in [6]. The article incorporates the constraint of the POD (Power Oscillation Damper) on the DFIG in the optimization model to the POD parameters to reduce the oscillation of both the DFIG and power systems. The test results verify the accuracy of the reduced model of the DFIG and validate the effectiveness of the improved optimization to the POD of the DFIG. The proposed method in [7] transforms the mathematical model of the system into a nonlinear ordinary differential equation form using Galerkin truncation and establishes an amplitude-frequency curve using the harmonic balance method and matrix analysis technique. The sensitivity analysis is performed using Newton's method on the analytical results of the amplitude-frequency curve. The article [8] discusses a new method for model order reduction of linear time-invariant passive systems, specifically positive real systems, using the BlTA concept. The method utilizes the phase angle of the transfer function for more accurate reduced-order models, which is a novel approach. The proposed algorithm involves the calculation of new gramians and associated Riccati equations and Lur'e equations using Kalman-Yakubovic-Popov lemma and linear matrix inequalities. This algorithm is shown to be a generalization of the positive real balanced truncation method and capable of providing more accurate approximation error. The paper [9] proposes a new method for implementing a discrete-time fractional-order proportional-integral-derivative (FOPID) controller. The method is based on a unique representation of the FOPID controller using a finite impulse response (FIR) filter to model the fractional properties. The balanced truncation model order reduction method is applied to obtain a low-order model of the FOPID controller. The methodology is extended to the controller with time-varying gains. The effectiveness of the methodology is confirmed in a real-life experiment involving the control of the DC motor servo system. The authors in [10] propose a new method for model order reduction of discrete-time systems using low-rank Gramian approximation. The method uses Laguerre functions expansions to calculate the low-rank decomposition factors of the controllability and observability Gramians. The reduced-order system is obtained through the low-rank square root method. A modified reduction procedure is also proposed to preserve stability in some cases. The authors in [11] develop a data-driven generalized BlTA based on two steps that rely on necessary and sufficient conditions for the common generalized Gramians of the systems. The proposed approach is illustrated by applying it to an example of a cart with a double-pendulum system. Additionally, the paper presents alternative procedures to compute upper bounds with respect to the true system generating the data.

To verify the ability to reduce the order of a high-order object model while retaining important physical properties of the original system and preserving system stability, as well as ensuring a small error between the reduced-order system and the original system, etc, the group of authors conducted research and applied the BlTA to reduce the order for robust control of synchronous generator load angle. The object applied was the stable part of order 22 of the RH stability controller in [12]. We used the BlTA to reduce the RH order to the 2nd, 3rd, and 4th orders. Simulations were carried out on Matlab, based on the results obtained from impulse response, frequency response, and the error between the original system and the reduced-order system, to provide comments and assessments on the applicability of the BlTA for robust control of synchronous generator load angle.

2. Balanced truncation algorithm (BlTA)

The BlTA is a widely used method in model order reduction. It is based on the principle that the most important dynamics of a system are those that can be excited and observed by the input and output, respectively [1]. The algorithm reduces the order of a high-dimensional model by projecting it onto a lower-dimensional subspace that captures these important dynamics. The key to the success of the BlTA is the construction of a balanced realization of the system, which is a state-space representation that is both controllable and observable and has balanced Gramians. This ensures that the projection preserves the most important dynamics of the system while minimizing the error between the reduced and original models. The BlTA has been applied in a wide range of fields, including control theory, signal processing, and electromagnetics. The BlTA method is described as follows [1]:

Input: The dynamical system asymptotic stable and minimally **RH(s)** is described by 4 state matrices (**A, B, C, D**) of order *n* as shown in expressions (1).

$$RH(s):\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)+Du(t)\end{cases}\Leftrightarrow RH(s):=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\Leftrightarrow RH(s):=C(sI-A)^{-1}B+D \quad (1)$$

where $A\in R^{n\times n},B\in R^{n\times m},C\in R^{p\times n},D\in R^{p\times m}$

- Step 1: Calculate **EP** and **EQ** satisfy the following linear matrix equation (Lyapunov equation): **EP** and **EP** are 2 positive definite matrices:

$$-BB^T=AEP+EA^T \quad (2)$$

$$-C^T=CA^TEQ+EQA \quad (3)$$

- Step 2: Cholesky decomposition of **EP** and **EQ**:

$$EP=UU^{\bullet} \quad (4)$$

$$EQ=VV^{\bullet} \quad (5)$$

- Step 3: SVD analysis:

$$V^{\bullet}U=NFM^{\bullet} \quad (6)$$

- Step 4: Calculate the transformation matrix:

$$Tr:=UMF^{-\frac{1}{2}}; \quad (7)$$

$$Tr^{-1}=F^{-\frac{1}{2}}N^{\bullet}V^{\bullet} \quad (8)$$

- Step 5: Convert to a balanced system:

$$Tr^{-1}AT=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$$

$$Tr^{-1}B=\begin{bmatrix}B_1\\ B_2\end{bmatrix} \quad (9)$$

$$CTr=\begin{bmatrix}C_1 & C_2\end{bmatrix}$$

where: $A_{11}\in R^{r\times r},A_{12}\in R^{r\times(n-r)},A_{21}\in R^{(n-r)\times r},A_{22}\in R^{(n-r)\times(n-r)},B_1\in R^{r\times m},B_2\in R^{(n-r)\times m},C_1\in R^{p\times r},C_2\in R^{p\times(n-r)}$

- Step 6: Choose the order r to reduce *r (r < n)*

Output: The dynamical system asymptotic stable and minimally **RHr(s)** is described by 4 state matrices (**A_r, B_r, C_r, D_r**) of order *r:* $(A_r,\ B_r,\ C_r,\ D_r)=(A_{11},B_1,C_1,D)$, where $A_r\in R^{r\times r},B_r\in R^{r\times m},C_r\in R^{p\times r},D_r\in R^{p\times m}$

3. Results and Discussion

In the paper [12], the authors introduced a Robust Controller RHinf for the Generator's Rotor Angle Stabilization Power System Stabilizer (PSS) System, with an order of 28, which posed difficulties in algorithm implementation, ensuring time reliability, and required powerful hardware configurations. We reduced the order of this robust controller to a lower order using BlTA, ensuring the criterion that the lower the order of the system, the better, with small error, and preserving the stability of the original system. Since BlTA can only reduce the order of stable systems, and RHinf is initially unstable, it needs to be separated into an unstable subsystem and a stable subsystem RH(s). Then, BlTA is applied to RH(s) with an order of 22 to reduce it to Rhr1(s) of order 2, Rhr2(s) of order 3, and Rhr3(s) of order 4. RH(s) is simulated with reduced-order systems Rhr1(s), Rhr2(s), and Rhr3(s) in the time domain (Figure 1) and frequency domain (Figure 2), and the error between the original system and the reduced-order systems is calculated. Finally, based on the results obtained, comments, evaluations, and recommendations are made on the application potential of BlTA in reducing the order of the high-order Robust Controller RHinf for the Generator's Rotor Angle Stabilization Power System Stabilizer (PSS) System.

$$RH(s)=\frac{R(s)}{H(s)}=\frac{\begin{array}{l}-92.88s^{22}-1.471e04s^{21}-1.09e06s^{20}-5.048e07s^{19}-1.647e09s^{18}-4.052e10s^{17}\\-7.854e11s^{16}-1.237e13s^{15}-1.616e14s^{14}-1.773e15s^{13}-1.64e16s^{12}-1.275e17s^{11}\\-8.284e17s^{10}-4.463e18s^{9}-1.976e19s^{8}-7.1e19s^{7}-2.042e20s^{6}-4.61e20s^{5}\\-7.952e20s^{4}-1.004e21s^{3}-8.658e20s^{2}-4.525e20s-1.073e20\end{array}}{\begin{array}{l}s^{22}+190.4s^{21}+1.711e04s^{20}+9.667e05s^{19}+3.862e07s^{18}+1.161e09s^{17}\\+2.73e10s^{16}+5.141e11s^{15}+7.878e12s^{14}+9.916e13s^{13}+1.03e15s^{12}+8.841e15s^{11}\\+6.254e16s^{10}+3.631e17s^{9}+1.718e18s^{8}+6.55e18s^{7}+1.98e19s^{6}+4.639e19s^{5}\\+8.165e19s^{4}+1.034e20s^{3}+8.82e19s^{2}+4.506e19s+1.036e19\end{array}}$$

$$RHr_1(s)=\frac{Rr_1(s)}{Hr_1(s}=\frac{-78.08s^2-197.8s-6184}{s^2+21.43s+597.6}$$

$$RHr_2(s)=\frac{Rr_2(s)}{Hr_2(s}=\frac{-97.87s^3-626.3s^2-9542s-8.412e04}{s^3+44.5s^2+518.8s+8128}$$

$$RHr_3(s)=\frac{Rr_3(s)}{Hr_3(s}=\frac{-93.22s^4-109.7s^3-1.074e04s^2-3.344e04s-1.021e05}{s^4+33.55s^3+449.1s^2+4471s+9863}$$

Absolute error and relative error according to H_∞ norm, between the original system RH(s) (22th order) and the reduced-order systems (2th, 3th and 4th orders) using the BlTA are shown in Table 1.

Table 1. *The error in H∞ norm between the original system and the reduced-order systems*

Order	$\|\|RH(s) - RHr(s)\|\|_{H\infty}$	$\|\|RH(s) - RHr(s)\|\|_{H\infty}/\|\|RH(s)\|\|_{H\infty}$
2	17.0815	0.1839
3	5.2925	0.0570
4	1.0799	0.0116

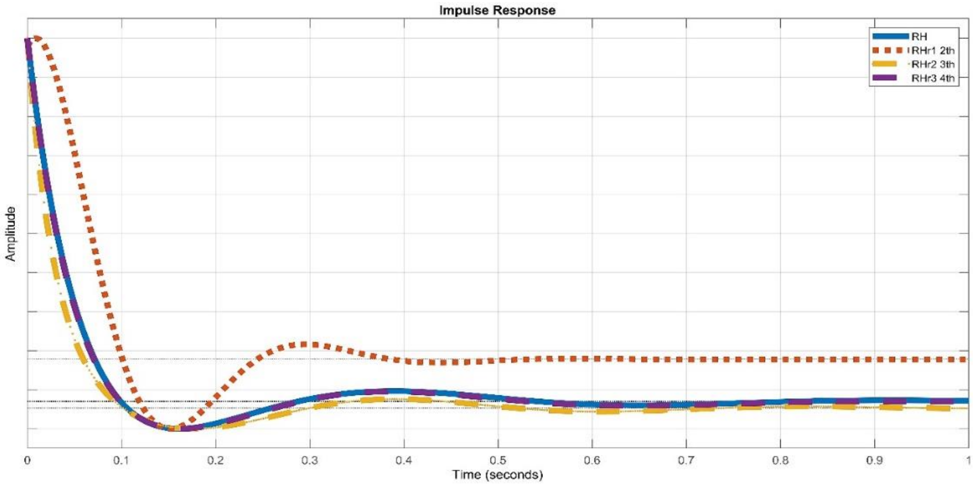

Figure 1. *Pulse responses of the original system and the reduced-order systems Rhr1(s), RHr2(s), RHr3(s)*

As shown in Table 1, the error of the reduced-order system decreases as the order of the system approaches the order of the original system. The absolute error indicates the maximum deviation between the original system and the reduced system in the time domain, while the relative error indicates the maximum deviation between the original system and the reduced system in the frequency domain. It is observed that the errors of the reduced systems in the frequency domain are much smaller than those in the time domain for the same reduced order. Therefore, the BlTA is effective in reducing the order of the robust controller of synchronous generator load angle and is suitable for application in the frequency domain.

Figure 1 shows the step response of the original system RH(s) and the reduced-order systems Rhr1(s) 2th, RHr2(s) 3th, RHr3(s) 4th using BlTA, within the simulation time range, it can be seen that: The second-order system has a response that differs greatly from the original RH(s) system. The third-order system is quite similar to the original RH(s) system, but still does not closely follow the amplitude-time response of the original system. The fourth-order system has a closely matched frequency response to the original RH(s) system, so it can be used as a replacement for the original 22nd-order system in the time response region.

Figure 2 shows the Bode plot of the original system RH(s) and the reduced-order systems Rhr1(s) 2th, RHr2(s) 3th, RHr3(s) 4th using BlTA, within the simulation frequency range, it can be seen that: A second-order system has a significantly different response compared to the original RH(s) system, it only closely matches the original system in the following ranges: Amplitude: from 0,1 (rad/s) to 0,5 (rad/s); Phase: from 30 (rad/s) to 2 (rad/s). A third-order system has a Phase-frequency plot that closely matches the original system, but its Amplitude response differs from the original system in the range of 0,5 (rad/s) to 22 (rad/s). Therefore the second-order and the third-order systems can be used to replace the original system in the frequency domain in the ranges where they closely match the original system. A fourth-order system has a Bode plot that matches the original RH(s) system, so it can be used to replace the original second-order system in the frequency response range.

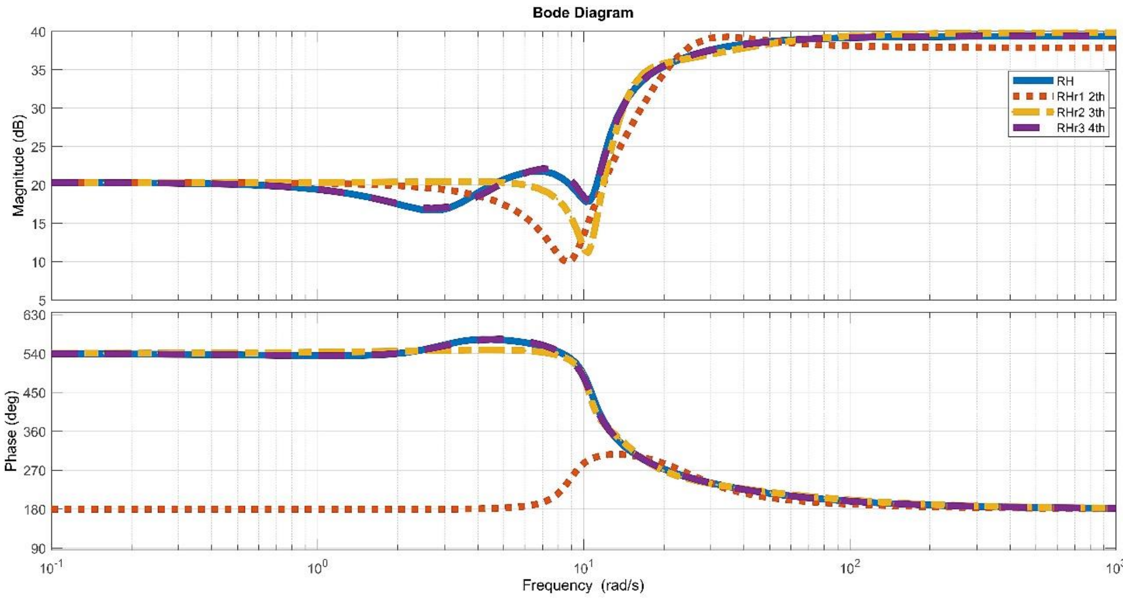

Figure 2. *Bode plot of the original system and the reduced-order systems Rhr1(s), RHr2(s), RHr3(s)*

4. Conclusion

Robust control theory aims to ensure stable performance in uncertain environments. The Balanced Truncation Algorithm is a method to reduce the model order of robust control for synchronous generators while preserving essential dynamics of the original system, providing accurate reduced-order models that are easy to implement and have been extensively developed. To verify the ability of BlTA to reduce orders for the robust controller of synchronous generator load angle, we conducted stability reduction of the controller to second, third, and fourth order. The simulation results on Matlab show that the fourth-order system closely matches the original system, with the smallest deviation from the original system, and can replace the original system in both time and frequency domains. The quality of the third and second-order systems is not high, but they still have a matching region with the original system, so within a small range, these two reduced-order systems can still be applied to replace the original system.

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