RESEARCH AND IMPLEMENTATION OF H-INFINITY BALANCED TRUNCATION ALGORITHM FOR HIGH-ORDER UNSTABLE SYSTEMS

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ABSTRACT

Unstable higher-order object models pose difficulties in analyzing and designing electrical systems due to their large dimensions and complexity. One solution to simplify these models is to reduce their order by eliminating higher-order dynamics. The H-Infinity balanced truncation (HBT) algorithm is a useful tool for addressing these challenges. The HBT algorithm reduces higher-order object models while retaining key dynamics of the original system by balancing energy across all modes. Using the HBT algorithm can simplify the analysis and design process and improve computational efficiency, leading to more accurate results and more efficient designs. To demonstrate the effectiveness of the HBT method, in this study, the algorithm was applied to a model of an unstable electrical system with a degree of 66 and reduced to degrees 8 and 15. Simulation results using Matlab showed that the HBT method was successful in reducing the order of the system and improved simulation time. The reduced-order system of degrees 8 or 15 can be used in place of the original system in applications in the time or frequency domain during analysis, design, simulation or implementation of electrical networks.

NGHIÊN CỨU VÀ TRIỂN KHAI THUẬT TOÁN CHẶT CÂN BẰNG H-INFINITY CHO HỆ THỐNG KHÔNG ỔN ĐỊNH BẬC CAO

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TỪ KHÓA

H-Infinity
Thuật toán chặt cân bằng H-Infinity
Hệ thống không ổn định bậc cao
Giảm bậc mô hình
Hệ thống điện cỡ lớn

TÓM TẮT

Các mô hình đối tượng bậc cao không ổn định gây khó khăn trong việc phân tích và thiết kế hệ thống điện do kích thước lớn và phức tạp của chúng. Một giải pháp để đơn giản hóa các mô hình này là giảm bậc của chúng bằng cách loại bỏ động lực bậc cao. Thuật toán cắt ngắn cân bằng H-Infinity (HBT) là một công cụ hữu ích để giải quyết các thách thức này. Thuật toán HBT giảm bậc các mô hình đối tượng bậc cao trong khi vẫn giữ lại các động lực chính của hệ thống gốc bằng cách cân bằng năng lượng trên tất cả các chế độ. Sử dụng thuật toán HBT có thể đơn giản hóa quá trình phân tích và thiết kế và cải thiện hiệu suất tính toán, dẫn đến kết quả chính xác hơn và thiết kế hiệu quả hơn. Để chứng minh tính hiệu quả của phương pháp HBT, trong bài báo này, thuật toán được áp dụng vào mô hình của một hệ thống điện không ổn định với bậc là 66 và giảm xuống các bậc 8 và 15. Kết quả mô phỏng bằng Matlab cho thấy rằng phương pháp HBT đã thành công trong việc giảm bậc của hệ thống và cải thiện thời gian mô phỏng. Hệ thống giảm bậc 8 hoặc 15 có thể được sử dụng thay cho hệ thống gốc trong các ứng dụng trong miền thời gian hoặc tần số trong quá trình phân tích, thiết kế, mô phỏng hoặc triển khai các mạng điện.

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1. Introduction

Higher-order systems are commonly used in various engineering fields. However, the complexity of such systems makes it difficult to analyze and control them effectively. Thus, it is necessary to reduce the model order to a lower order without losing essential system properties. Model order reduction (MOR) has become a crucial technique to achieve this goal. Many MOR algorithms have been developed, such as Balanced Truncation [1] and Modal Reduction [2], etc. However, these algorithms are limited to stable systems.

In contrast, the H-infinity balanced truncation (HBT) algorithm [3] reduce the order of unstable higher order systems directly. Therefore, the H-Infinity balancing algorithm has gained significant attention in recent years. Paper [4] discusses the HBT and Optimal Projection methods for reduced-order robust control. The fourth Optimal Projection equation is found to have a crucial role in guaranteeing a minimized auxiliary cost and removing nonminimal controller states when applied to the full-order case. Although the discussion is focused on the normalized H_∞ problem, the arguments can be applied to more general H_∞ control problems. The Optimal Projection method can also be extended to cover control problems with different H_∞ and LQG objectives. The article [5] discusses the use of HBT for gaining reduced-order plants or controllers. The method involves compensating an unstable plant using a specific robust stabilizing controller and then using the two Riccati equations to define a set of closed-loop input-output invariants called H_∞-characteristic values. These values are then used to discard the corresponding part of the plant or controller to obtain a reduced-order system. The article describes an easily testable criterion for determining the ability of the reduced-order controller to stabilize the full-order plant. Additionally, an upper bound on the H_∞-norm of the closed-loop is derived based on the prespecified H_∞-norm bound and the discarded H_∞-characteristic values. The method is computationally simple and can be applied to symmetric passive systems. The article suggests the possibility of using loop-shaping ideas to apply the normalized approach to shaped plants, which could be an interesting area for future research. The authors in [6] give an H_∞ error bound for the difference of a positive real transfer function and its positive real real real balanced truncation. The author provides a correct error bound for the difference of a proper positive real transfer function and its positive real real balanced truncation, as well as an error bound in the gap metric. The paper [7] proposes a new method of H controller order reduction based on minimal loss of information theory and a balanced truncation algorithm. The method calculates a new controllability grammian matrix and obtains a reduced H controller. Simulation results demonstrate the effectiveness of the proposed method in enhancing the performance of the closed-loop system with low system error bonds. Comparative analysis with two other methods supports the feasibility of the proposed approach. Research work [8] propose the design of a centralized interaction-independent controller for a two-input fourth-order integrated (TIFOI) dc-dc converter using an H-infinity Loop-shaping design procedure. The converter Regulates dc-bus voltage and low voltage source current with two switching devices and four energy storage elements. Interactions between the output variables of the system must be measured for proper controller structure selection. To overcome ambiguity in selecting suitable controller architectures, the H-infinity Loop-shaping design procedure was used to design a centralized controller that robustifies loop characteristics of the converter system by minimizing the infinity norm associated with the set of four cost functions. The higher order controllers were reduced to lower order using balanced truncation model reduction. The load division and bus-voltage regulation features of the TIFOI dc-dc converter were demonstrated through simulation on a prototype, 36 V/12 V to 24 V, 100 Watt, dc power distribution system. The analytical design studies were verified in simulation in the PSIM environment. The paper [9] explores closed-loop order reduction techniques for model-based control designs like H2 and H∞ optimal control. The paper proposes two algorithms to reduce the loss of performance resulting from truncation, which can be applied to a large class of controller designs and order reduction techniques. The algorithms minimize the performance loss as a result of truncation over all subsets of a given cardinality but do not guarantee that the performance loss is minimized. The paper also shows that the magnitude of LQG singular values does not determine the loss of performance as a result of truncation, which has implications for a larger class of order reduction techniques. The paper [10] proposes a new strategy for optimal frequency regulation in a power system consisting of regional AC grids and an offshore wind farm linked by a multi-terminal high voltage direct-current (MTDC) network. Decentralized H_∞ controllers are developed to coordinate the operations of AC synchronous generators and hybrid MTDC converters, ensuring optimal power sharing and minimizing frequency deviations in each grid. Robust optimization problems are formulated to develop the decentralized H_∞ controllers, considering parameter uncertainty and decentralized control inputs and outputs. Model orders of the resulting controllers are reduced using balanced truncation to eliminate unobservable and uncontrollable state variables while preserving their dominant response characteristics. Sensitivity and eigenvalue analyses are conducted to investigate the effects of various factors, such as parameter uncertainty, communication delays, and measurement errors. Comparative case studies demonstrate that the proposed strategy is more effective, stable, and robust in reducing frequency deviations in MTDC-linked grids than conventional strategies, even under various challenging conditions such as communication failures and different weighting functions.

Thus, researchers still focus on using HBT for MOR, with recent studies focusing on improving and adjusting algorithms or directly applying methods to specific problems, objects, or systems requiring specific order reduction. In order to verify and evaluate HBT methods, the authors applied them to an object model, an unstable power system in [11], with an order of 66, to reduce it to order 10 and 25. They simulated responses in the time and frequency domains and calculated errors between the original and reduced order systems to make observations on the effectiveness of these algorithms for the considered object in terms of order reduction ability.

2. H-Infinity balancing algorithm

The algorithm H-infinity balanced truncation (HBT) [3] is presented as follows:

Input: The dynamical system G(s) is described by 4 state matrices (A, B, C, D) of order n. Where: $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}, D \in \mathbb{R}^{p\times m}$

- Step 1: Solve two Riccati equations to determine the control Gramian P and the observation Gramian Q: (P > 0; Q > 0).

$$AP + PA^T + BB^T = \left(1-\beta^{-2}\right)\left(PC^T + BD^T\right)\left(I + DD^T\right)^{-1}\left(PC^T + BD^T\right)^T \tag{1}$$

$$A^TQ + QA + C^TC = \left(1-\beta^{-2}\right)\left(B^TQ + D^TC\right)^T\left(I + D^TD\right)^{-1}\left(B^TQ + D^TC\right) \tag{2}$$

where β ($0 < \beta \le 1$) is the optimal cost computed during the construction of the H-Infinity controller.

- Step 2: Cholesky decomposition of P and Q as follows:

$$P = JJ^\top \tag{3}$$

$$Q = KK^\top \tag{4}$$

- Step 3: SVD analysis:

$$K^\top J = USV^\top \tag{5}$$

- Step 4: Calculate the transformation matrix:

$$Tcv := JVS^{-\frac{1}{2}}; \tag{6}$$

$$Tcv^{-1} = S^{-\frac{1}{2}}U^\top K^\top \tag{7}$$

- Step 5: Calculate Balance transformation:

$$\left[Tcv^{-1}AT, Tcv^{-1}B, CTcv, D\right] = \left[\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \begin{bmatrix} C_1 & C_2 \end{bmatrix}, D\right] \tag{8}$$

- Step 6: Choose the order r to reduce *r (r < n)*

Output: The reduced-order system of order *r:* $\left(A_r, B_r, C_r, D_r\right) = (A_{11}, B_1, C_1, D)$, where $A_r \in \mathbb{R}^{r\times r}, B_r \in \mathbb{R}^{r\times m}, C_r \in \mathbb{R}^{p\times r}, D_r \in \mathbb{R}^{p\times m}$.

3. Results and Discussion

The literature [11] presents a higher-order power system model known as the New England Actual Electrical System (CEPEL, Brazil). This benchmark can be accessed through the data file *ww_36_pmec_36.mat*, and it is a single-input, single-output system with an order of 66, consisting of 4 matrices A, B, C, D. It should be noted that this system is unstable due to the presence of a pole with a positive real part.

The HBT method is applied to reduce the complexity of this unstable 66th-order electric system using lower-order equivalent systems. The reduced orders are 8th and 15th. The algorithms are implemented and simulated on Matlab, resulting in the time-domain and frequency-domain responses shown in Figure 1 and Figure 2, respectively.

Figure 1 shows the step response of the original system (66th order: G(s)) and the reduced-order systems (8th order: Gr1(s), 15th order: Gr2(s) using HBT algorithms, within the simulation time range, it can be seen that: Both step responses of the reduced-order systems with orders 8 and 15 closely match with the original system. Therefore the reduced-order systems with orders 8 and 15 can be used as replacements for the original system when working with responses in the time domain.

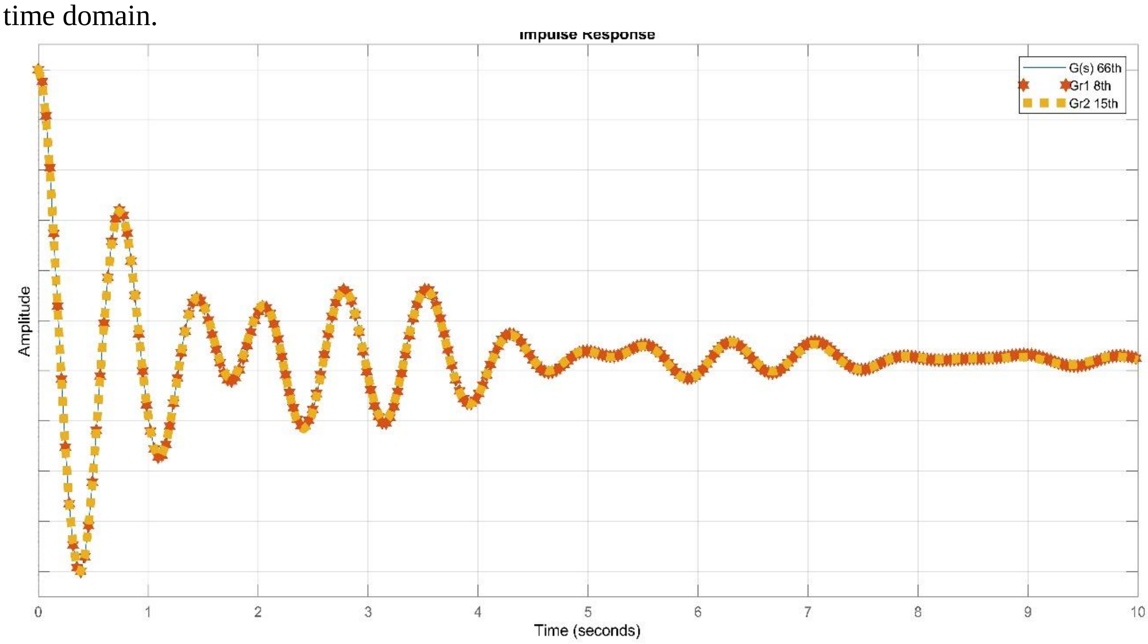

Figure 1. *Pulse responses of the original system and the reduced-order system of 8th and 15th orders*

Figure 2 shows the Bode plot of the original system (66th order: G(s)) and the reduced-order systems (8th order: Gr1(s), 15th order: Gr2(s) using HBT algorithms, within the simulation frequency range, it can be seen that:, it can be seen that: The frequency response of the reduced-order system with order 8 approximates the original system and only matches with the original system within the range of $2.5\text{x}10^{-5} \div 0.5$ (rad/s). The phase angle (deg) and magnitude (dB) responses of the reduced-order system with order 15 match with the original system. Therefore the reduced-order system with order 15 can be used as a replacement for the original system when working with responses in the frequency range of $10^{-6} \div 10^{2}$ (rad/s).

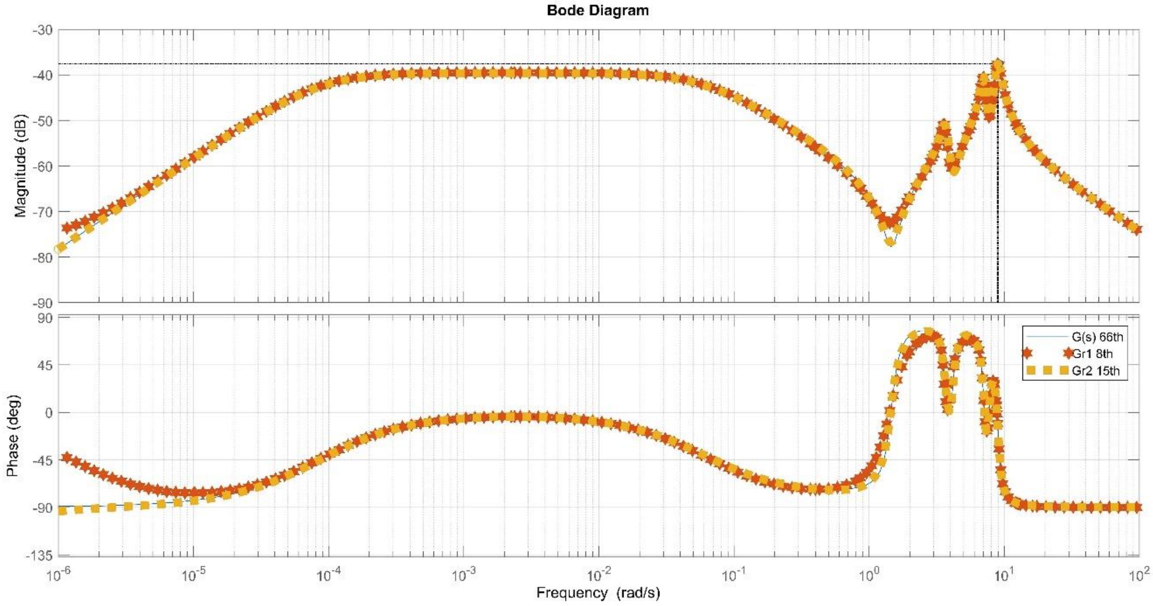

Figure 2. *Bode plot of the original system and the reduced systems of 8th and 15th orders*

Absolute error and relative error according to H_∞ norm, between the original system (66th order) and the reduced-order systems (8th and 15th orders) using the HBT method are shown in Table 1.

Table 1. *The error in H_∞ norm between the original system and the reduced-order systems*

ORDER	$\|G(s) - Gr(s)\|_{H\infty}$	$\|G(s) - Gr(s)\|_{H\infty}/\|G(s)\|_{H\infty}$
8	0.001158424388902	0.087626959215432
15	$3.581280971890707 \times 10^{-05}$	0.002708996501364

As shown in Table 1, the error of the reduced-order system decreases as the order of the system approaches the order of the original system. Conversely, the simulation time becomes faster as the order of the system decreases. Specifically, the simulation time of the original system, the system reduced to the 15th and the 8th order, respectively, is 0.0074 (seconds); 0.0060 (seconds) and 0.0033 (seconds).

4. Conclusion

Unstable higher-order object models can present significant challenges in the analysis and design of electrical systems. These models may have large dimensions and can be difficult to manipulate and process. One approach to addressing this issue is to reduce the order of the model, which involves simplifying the model by removing the higher-order dynamics and can directly reduce the order of the unstable system. To overcome these challenges, the H-Infinity balanced truncation algorithm has been developed. This algorithm allows for the reduction of higher-order object models unstable while ensuring that the reduced model retains the key dynamics of the original system. The algorithm achieves this by balancing the energy of the system across all modes, ensuring that the energy is conserved during the truncation process.

The use of the HBT algorithm has significant implications for the analysis and design of electrical systems. By reducing the order of higher-order object models, the algorithm can simplify the analysis and design process while still retaining the key dynamics of the system.

This can lead to more accurate results and more efficient designs. Additionally, the algorithm can improve the computational efficiency of the analysis process, reducing the time and resources required to perform simulations and analysis. The HBT algorithm is a powerful tool for the descaling of unstable higher-order object models. By addressing the challenges associated with higher-order models, the algorithm can improve the accuracy and efficiency of the analysis and design process, leading to better designs and more efficient systems.

To verify the ability of the HBT method to reduce the order of a system, the authors applied this algorithm to the model of an unstable electrical system with a degree of 66, to reduce it to degrees 8 and 15. From the simulation results on Matlab between the original system and the reduced-order system, it can be seen that the HBT method is effective and simulation time is faster when the system order is reduced. During the analysis, design, simulation or implementation of electrical networks, one can use the reduced-order system of degrees 8 or 15 instead of the original system in applications in the time or frequency domain. The next development direction of the authors is to find the optimal order to be reduced based on one or more criteria to be preserved in the original system, or to evaluate and compare other order reduction algorithms with the HBT method. In addition, the authors also study new methods to better meet the requirements of model order reduction.

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