RESEARCH AND APPLICATION OF THE OPTIMAL HANKEL NORM APPROXIMATION ALGORITHM FOR REDUCING THE ORDER OF HIGH-ORDER MIMO SYSTEMS

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ABSTRACT

The Optimal Hankel Norm Approximation (OHkNA) algorithm plays an important role in reducing the order of the Multi-Input Multi-Output (MIMO) systems. By reducing the order of the system, the OHkNA algorithm simplifies the system's design and performance. One of the advantages of OHkNA is its ability to achieve a good balance between the reduction of the system order and the preservation of its important features. In order to verify its effectiveness, we have studied and applied this algorithm to a 4-input, 4-output MIMO system of order 14, and the simulation results were analyzed. The results indicate that the system reduces to order 8 and exhibits good response in both time and frequency domains. Additionally, the error of order reduction is very small, indicating that the Optimal Hankel Norm Approximation algorithm is an efficient method for reducing the order of the MIMO system. Overall, the algorithm's ability to reduce the order of the MIMO system while maintaining system performance makes it a valuable tool in the field of control engineering, signal processing, simulation, and communication systems.

NGHIÊN CỨU VÀ ÁP DỤNG THUẬT TOÁN XẤP XỈ CHUẨN HANKEL TỐI ƯU ĐỂ GIẢM BẬC CHO HỆ MIMO BẬC CAO

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TỪ KHÓA

Chuẩn Hankel
Thuật toán xấp xỉ chuẩn Hankel tối ưu
Hệ thống bậc cao
Giảm bậc mô hình
Hệ thống đa đầu vào-đa đầu ra bậc cao

TÓM TẮT

Thuật toán Xấp xỉ chuẩn Hakel tối ưu (OHkNA) đóng một vai trò quan trọng trong việc giảm bậc của các hệ Đa đầu vào-Đa đầu ra (MIMO). Việc giảm bậc của hệ thống bằng OHkNA giúp đơn giản hóa thiết kế và tăng hiệu suất của hệ thống. Một trong những ưu điểm của OHkNA là khả năng đạt được sự cân bằng tốt giữa việc giảm bậc hệ thống và bảo tồn các đặc điểm quan trọng của nó. Để xác nhận tính hiệu quả của OHkNA, chúng tôi đã nghiên cứu và áp dụng thuật toán này vào một hệ MIMO 4 đầu vào, 4 đầu ra bậc 14 và phân tích kết quả mô phỏng. Kết quả cho thấy hệ thống giảm bậc xuống 8 và cho thấy phản hồi tốt cả ở miền thời gian và tần số. Ngoài ra, sai số giảm bậc rất nhỏ, cho thấy thuật toán OHkNA là một phương pháp hiệu quả để giảm bậc của hệ MIMO. Tổng thể, khả năng giảm bậc của hệ MIMO của thuật toán OHkNA trong khi vẫn giữ được hiệu suất hệ thống khiến nó trở thành một công cụ quý giá trong lĩnh vực kỹ thuật điều khiển, xử lý tín hiệu, mô phỏng và hệ thống truyền thông.

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1. Introduction

In the field of signal processing and control engineering, model order reduction (MOR) is a technique that is widely used to reduce the complexity of a system without compromising its performance. In the context of higher order MIMO (multiple-input, multiple-output) systems, model order reduction involves reducing the order of a system's transfer function by eliminating the high-order dynamics that are not significant for the system's behavior.

A higher order MIMO system typically consists of multiple input and output channels, each with its own dynamics and interdependencies. Such systems are commonly used in a wide range of applications, such as wireless communication, control systems, and image processing. However, as the order of the system increases, the complexity of the system also increases, making it more difficult to analyze and control.

Model order reduction has several advantages for higher-order MIMO systems. First, it reduces the computational complexity of the system, making it easier to analyze and control. Second, it can improve the stability and performance of the system by removing the insignificant dynamics that may cause instability or oscillations. Third, it enables faster simulation and real-time control of the system by reducing the computation time required to evaluate the system's behavior. The choice of the appropriate model order reduction technique depends on the specific characteristics of the system, such as its size, complexity, and dominant dynamics. Overall, model order reduction is a powerful tool for engineers and researchers working with higher order MIMO systems, enabling them to achieve better control and analysis of complex systems.

Model order reduction for higher order MIMO systems can be achieved using various techniques, including balanced truncation, singular perturbation, and Krylov subspace methods. These techniques aim to identify the dominant dynamics of the system and remove the insignificant dynamics, thus reducing the order of the system's transfer function. The goal of model order reduction is to obtain a simplified model that accurately captures the system's behavior while reducing the computational complexity of the system.

Balanced truncation is a popular technique for model order reduction in higher order MIMO systems [1]. This method involves computing a balanced realization of the system, which is a transformation of the original system that balances the controllability and observability Gramians. The balanced realization is then truncated by discarding the high-order dynamics, resulting in a reduced-order model that retains the dominant dynamics of the system.

Singular perturbation is another approach for model order reduction that is suitable for higher order MIMO systems with multiple time scales [2]. This method involves separating the fast and slow dynamics of the system and reducing the order of the slow dynamics, while preserving the fast dynamics. The result is a reduced-order model that accurately captures the slow dynamics of the system while neglecting the fast dynamics.

Krylov subspace methods are a family of iterative methods that can be used for model order reduction in higher order MIMO systems [3]. These methods involve constructing a Krylov subspace matrix from the system's transfer function and using it to compute a low-order approximation of the system's transfer function. This technique is particularly useful for large-scale systems with sparse matrices, as it avoids the computation of the full system matrix.

Another approach for model order reduction in higher-order MIMO systems is the Optimal Hankel Norm Approximation (OHkNA) algorithm [4]-[9]. This method involves constructing a Hankel matrix from the system's impulse response and using it to obtain a low-order approximation of the system's transfer function. The OHkNA algorithm aims to minimize the error between the original and reduced-order models by finding the optimal weighting matrix that balances the error in the input and output signals. The OHkNA algorithm is particularly suitable for MIMO systems with a high degree of interdependence between the input and output channels. This method can capture the essential dynamics of the system while reducing its order, resulting in a simplified model that accurately represents the system's behavior. Engineers and researchers working with MIMO systems can benefit from the OHkNA algorithm to achieve better control and analysis of complex systems.

Because of its effective ability to reduce the order of higher-order object models, the OHkNA algorithm continues to be researched, improved, and developed. It can be found in research works and scientific articles such as: The authors in [10] presents an example of a technique for reducing the order of highly unstable systems, such as the NASA HIGHMAT FIGHTER, using the OHkNA. To address the challenges of highly unstable systems, a decomposition approach has been applied to separate the system into its unstable and stable components. The OHkNA has been employed to obtain the stable part of the reduced-order model. The reduced model of the stable part is combined with the unstable part of the decomposition to develop an overall reduced model system. The effectiveness of this technique is demonstrated through mathematical analysis of the plant. In the study [11], the authors have employed a unique approach that utilizes an extension of the Adamyan-Arov-Krein theorem to design digital filter banks. This algorithm facilitates the optimal reduction of a reference filter bank's order in the Hankel norm. The study includes a summary of the relevant mathematical background, followed by an outline of the numerical implementation. The authors present an example that demonstrates the design of a bank of bandpass filters, which can effectively split a signal into frequency bands. The paper [12] proposes Gramians-based frequency-weighted model order reduction frameworks for discrete-time one-dimensional and two-dimensional systems. This approach provides an easily measurable a priori error-bound expression compared to other stability-preserving techniques. The objective of this study [13] is to improve the performance of a Pressurized Heavy Water Reactor operating under a step-back condition through the design of a Fractional Order Proportional Integral Derivative controller. The OHkNA method is employed to obtain a Reduced Order Model for this system. The unknown controller parameters are optimized using the Nelder-Mead algorithm. Performance analysis is conducted to demonstrate the effectiveness of the designed controller. The article [14] proposes a robust control method for the BPC in the AC/DC hybrid microgrid based on the H_∞ tracking problem. To solve the voltage control problem, the paper uses the superposition theorem to transform it into an H_∞ tracking problem and proves the stability of a robust controller. The paper then reduces the design H_∞ robust controller using the OHkNA reduction method.

To validate the OHkNA method's dimensionality reduction capability, the authors applied this algorithm to reduce the order of a 4-input 4-output MIMO system with an order of 14, as described in [15], to an order of 8. We then simulated the reduced-order system and compared its time-domain response (impulse response) and frequency-domain response (magnitude response) with the original system using Matlab. Finally, we calculated the error between the original and reduced-order systems and drew conclusions based on the results.

2. Optimal Hankel norm approximation algorithm

According to the document [4]-[9], the Optimal Hankel Norm Approximation (OHkNA) algorithm is the problem of finding the approximation $\hat{G}(s)$ of McMillan degree $r < n$ such that the norm of the error $\| G(s) - \hat{G}(s) \|_H$ is minimized. Assuming the eigenvalues of the matrix **A** lie to the left of the complex plane, the control Gramian **P** and the observability Gramian **Q** are defined as follows:

$$P = \int_0^\infty e^{At} B B^T e^{A^T t} dt \tag{1}$$

$$Q = \int_0^\infty e^{A^T t} C^T C e^{At} dt \tag{2}$$

where **P** and **Q** satisfy the following linear matrix equation (Lyapunov equation):

$$BB^T = -AP - PA^T \tag{3}$$

$$C^T = -CA^T Q - QA \tag{4}$$

The Hankel norm of the stable system transfer function **G(s)** is defined by:

$$\| G(s) \|_H = \lambda_{\max}^{1/2}(PQ) \tag{5}$$

where λ_{max} is the largest eigenvalue of the **PQ** product matrix. It is a measure of the energy of the controllable and observable states of the system, and is crucial for model order reduction. Its main goal is to eliminate unimportant states from the input-output perspective, i.e., states that are less controllable/observable. The singular values of the stable system are the square roots of the eigenvalues of the **PQ** product matrix. They indicate the energy of the corresponding states of the system. Typically, these singular values are arranged in decreasing order along the main diagonal: $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq \sigma_{r+1} \geq \cdots \geq \sigma_n > 0$

The upper bound of the error using the OHkNA method is

$$\| G(s) - \hat{G}(s) \|_H \geq \sigma_{r+1} \tag{6}$$

For the balanced system (***Ab, Bb, Cb, Db***) of ***G(s)***, satisfy:

$$Ab = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ Bb = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},\ Cb = \begin{pmatrix} C_1, & C_2 \end{pmatrix}, Db = D \tag{7}$$

Calculate the matrices:

$$\mathrm{m} = \mathrm{S}_1^2 - \sigma_{r+1}^2 I \tag{8}$$

$$V = -C_2 \left(B_2^T \right)^{\perp} \tag{9}$$

$$\begin{aligned} Ar &= m^{-1}\left(\sigma_{r+1}^2 A_{11}^T + \Sigma_1 A_{11} S_1 - \sigma_{r+1} C_1^T V B_1^T\right), \\ Br &= m^{-1}\left(S_1 B_1 + \sigma_{r+1} C_1^T V\right), \\ Cr &= C_1 S_1 + \sigma_{r+1} V B_1^T, \\ Dr &= D + \sigma_{r+1} V \end{aligned} \tag{10}$$

where $\left(B_2^T\right)^{\perp}$ is the pseudo-inverse of $\left(B_2^T\right)$.

Then the r-order reduction system is:

$$Gr(s) = Cr(sI - Ar)^{-1} Br + Dr \tag{11}$$

3. Results and Discussion

Microgrid (MG) system with 4 inputs and 4 outputs consisting of two inverter-interfaced microsources (MSs) with a dedicated RLC load connected at the point of common coupling (PCC) [15]. The loads are tunable and can replicate any standard electrical or electronic equipment. During islanded operation, the inverters operate in the voltage controlled model (VCM) and are connected to the LV network through a series filter. It is assumed that each MS is a dispatchable source or has adequate storage units to maintain a constant DC-link voltage at the input side of the inverter. The higher order system model of the standard MG, as derived from its state-space model, would create challenges for controller design and stability studies, particularly with an increase in the number of devices leading to further complexity.

Applying the OHkNA algorithm to reduce the order of this 14th order MIMO system to 8th order, and then simulating it on Matlab, we obtain the time-domain response as shown in Figure 1, and the frequency-domain responses as shown in Figure 2.

Figure 1 shows that the step response of the original system and the 8th reduced-order system using OHkNA algorithms, within the simulation time range, can be seen: The step responses from input 3 to output 1, from input 3 to output 2, from input 4 to output 1, and from input 4 to output 2 in the time interval from 0.5 seconds to 0.8 seconds differ from the original system, while the other input-output pairs have matching responses with the original system. Therefore, the reduced-order system of order 8 can be used to replace the original system in time-domain applications.

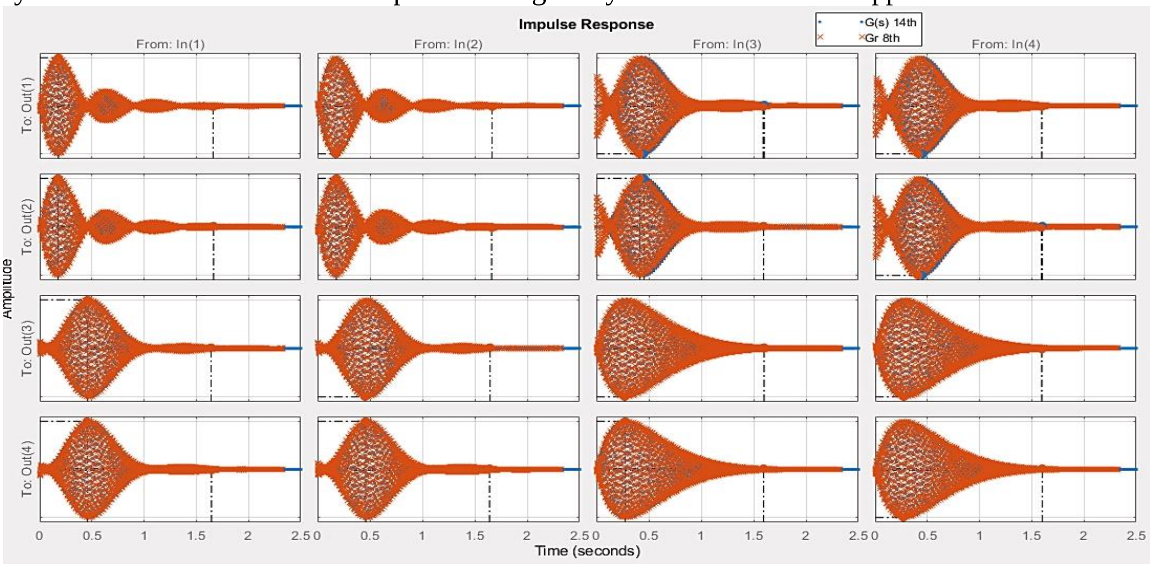
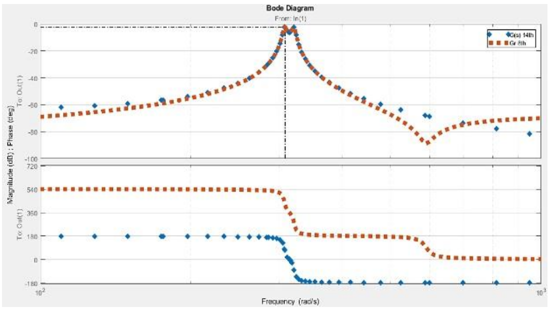
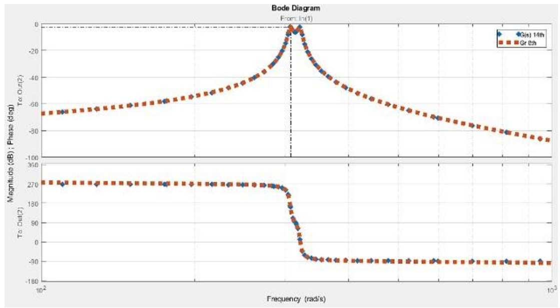


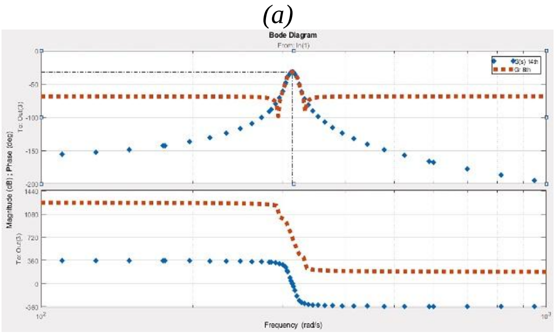
Figure 1. *Pulse responses of the original system and the reduced-order system of 8th*



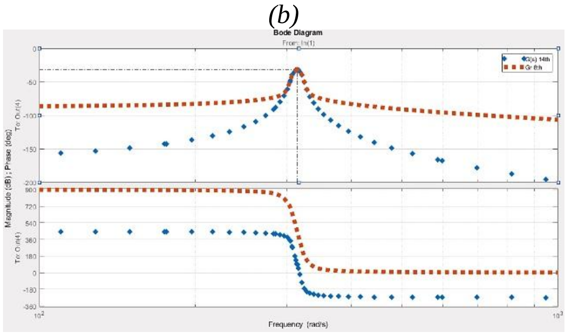
(a)



(b)

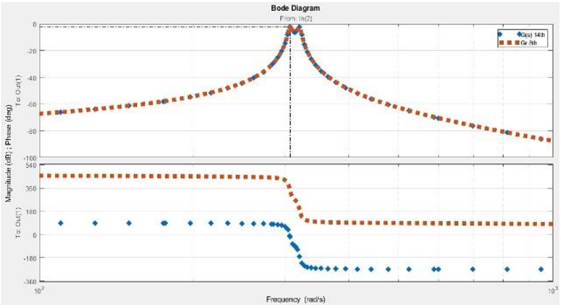


(c)

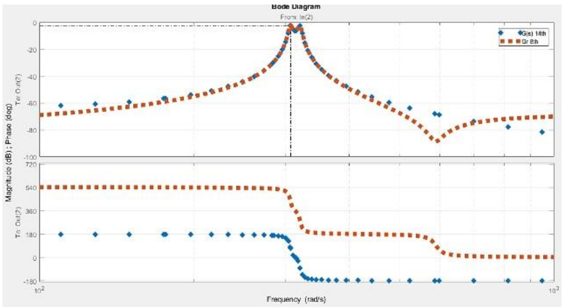


(d)

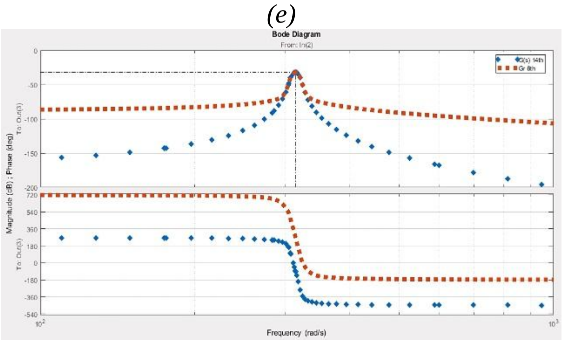
Figure 2. *The magnitude response in the frequency domain between the original system and the reduced-order system of order 8: (a) From In(1) to Out(1); (b) From In(1) to Out (2); (c) From In(1) to Out(3); (d) From In(1) to Out (4)*



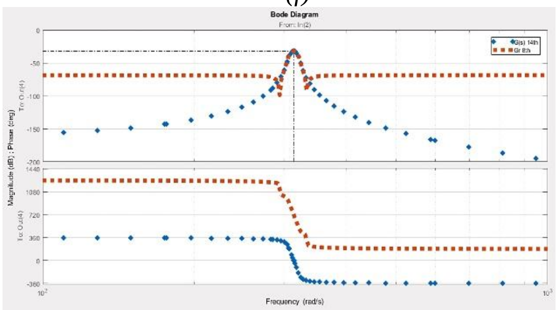
(e)



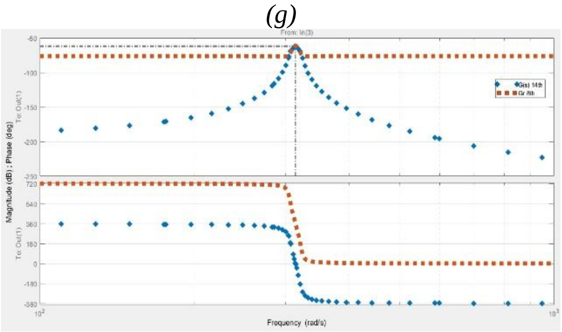
(f)



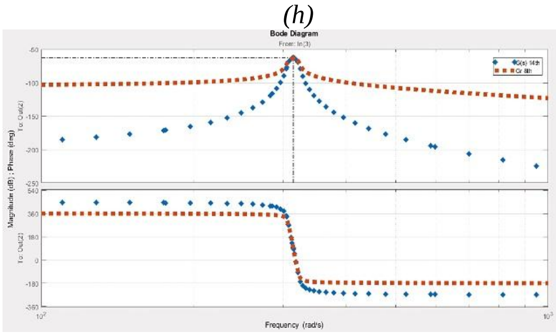
(g)



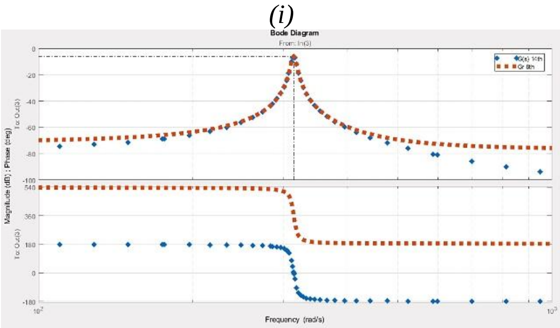
(h)



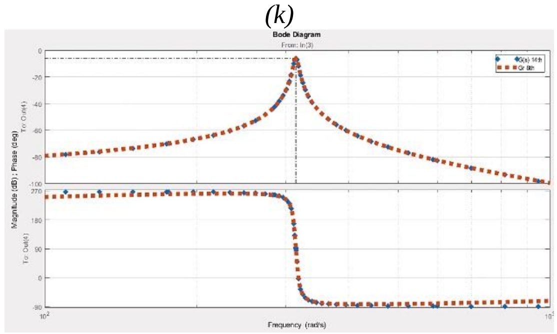
(i)



(k)



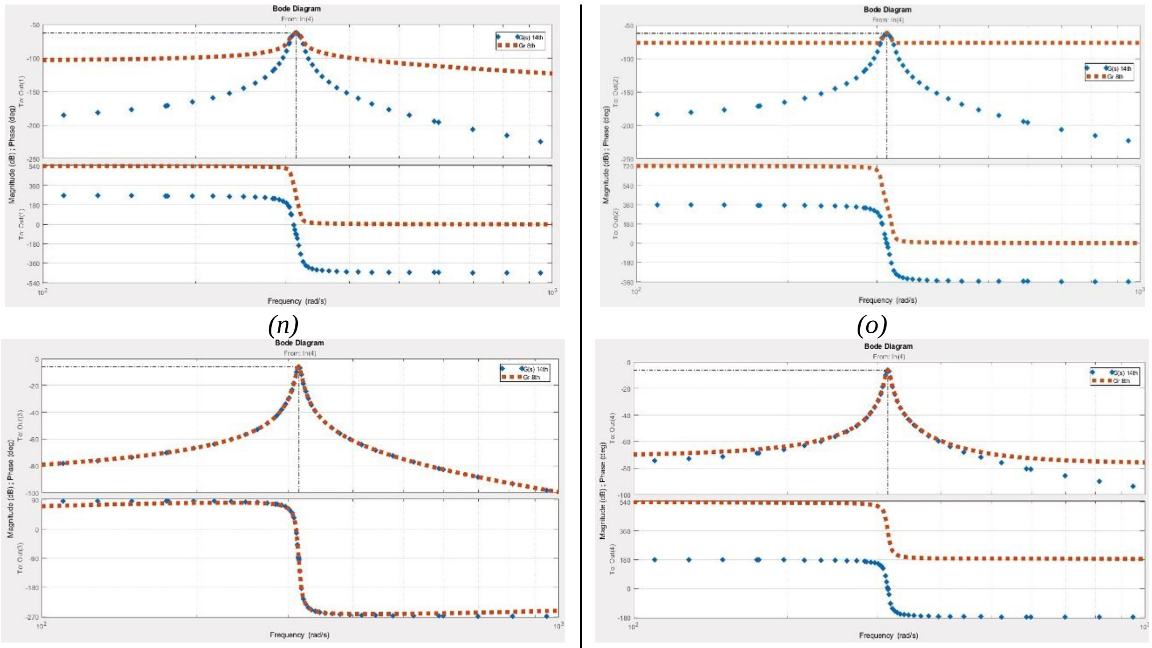
(l)



(m)

Figure 2. *The magnitude response in the frequency domain between the original system and the reduced-order system of order 8:*
(e) From In(2) to Out(1); (f) From In(2) to Out (2); (g) From In(2) to Out(3); (h) From In(2) to Out (4); (i) From In(3) to Out(1); (k) From In(3) to Out (2); (l) From In(3) to Out(3); (m) From In(3) to Out (4)

Figure 2 shows the Bode plot of the original system and the 8th reduced-order system using OHkNA algorithms, within the simulation time range, it can be seen that: The magnitude responses in the frequency domain between the original system and the reduced-order system match each other at: from input 1 to output 2 in Figure 2(b), from input 2 to output 1 in Figure 2(e), from input 3 to output 4 in Figure 2(m), and from input 4 to output 3 in Figure 2(p). Therefore, it is entirely possible to reduce the order of the system to 8 and use the reduced-order system instead of the original 14th-order system in applications related to magnitude responses in the frequency domain. The magnitude response in the frequency domain of the original system deviates from that of the 8th-order reduced system at the input-output pairs. They only match in the working regions: frequency in the range of 250 (rad/s) to 400 (rad/s) and magnitude in the range of -75(dB) to -45(dB). Therefore, only the 8th-order reduced system can be used instead of the original 14th-order system in applications related to magnitude response in this working region.



(n) (o)

(p) (q)

Figure 2. *The magnitude response in the frequency domain between the original system and the reduced-order system of order 8:*
(n) From In(4) to Out(1); (o) From In(4) to Out (2); (p) From In(4) to Out(3); (q) From In(4) to Out (4)

Absolute error and relative error between the original system and the reduced order system respectively indicate the maximum deviation in the entire working region over the time domain and the frequency domain. Absolute error and relative error according to H_∞ norm, between the original system (14th order) and the reduced-order system (8th order) using the OHkNA are $5.910966291206777 \times 10^{-4}$ and $3.915485271769241 \times 10^{-4}$ respectively.

4. Conclusion

The Optimal Hankel Norm Approximation OHkNA algorithm plays a crucial role in the order reduction of Multi-Input Multi-Output (MIMO) systems. This algorithm aims to reduce the system's order while maintaining its transfer function. It is a well-established technique used in the field of control systems and signal processing. By reducing the system's order, it becomes less complex, which makes it easier to analyze and implement. This is particularly important for MIMO systems, which often have a high number of inputs and outputs.

To evaluate the effectiveness of the OHkNA algorithm in reducing system order, the authors of a recent study applied it to a 4-input, 4-output MIMO system. The simulation results demonstrated that the system could be reduced to order 8 while maintaining a good response in both time and frequency domains. Moreover, the error of order reduction was negligible, which indicates that the OHkNA algorithm is capable of providing a high degree of accuracy.

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