MIXED-INTEGER LINEAR PROGRAMMING-BASED TRANSMISSION NETWORK EXPANSION PLANNING CONSIDERING POWER LOSS

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ABSTRACT

Transmission network expansion planning (TNEP) is an important problem in power system planning. This problem is usually modelled as a mixed-integer nonlinear programming (MINLP) formulation. The MINLP model, however, often does not guarantee a globally optimal solution and is not likely to be computationally efficient. This paper proposes a mixed-integer linear programming (MILP) model aiming to optimize the topology of transmission networks considering power loss. The proposed MILP model is converted from the MINLP model using the piecewise linearization technique and precisely linearizing the product of the binary and continuous variables. The objective function of the TNEP problem in this study is to minimize the total cost of the power system, including the investment cost for new transmission lines, maintenance cost, cost related to energy losses loss, and production cost of generators. The considered constraints comprise the investment budget limit, the system of power flow equations, the power flow limits of branches and the output limits of generators. The evaluation of the proposed MILP model is implemented on an IEEE 24-node network using a commercial CPLEX optimizer with GAMS programming language.

QUY HOẠCH MỞ RỘNG LƯỚI ĐIỆN TRUYỀN TẢI CÓ XÉT TỔN THẤT CÔNG SUẤT SỬ DỤNG QUY HOẠCH TUYẾN TÍNH NGUYÊN THỰC HỖN HỢP

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TỪ KHÓA

Quy hoạch mở rộng lưới điện truyền tải (TNEP)
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Tổn thất điện năng
Tuyến tính hóa từng đoạn

TÓM TẮT

Quy hoạch mở rộng lưới điện truyền tải (TNEP) là một nhiệm vụ quan trọng trong quy hoạch hệ thống điện. Bài toán này thường được mô hình dưới dạng quy hoạch phi tuyến nguyên thực hỗn hợp (MINLP). Tuy nhiên, mô hình MINLP thường không đảm bảo nghiệm tối ưu toàn cục và không hiệu quả về mặt tính toán. Bài báo này đề xuất mô hình quy hoạch tuyến tính nguyên thực hỗn hợp (MILP) để tối ưu hóa cấu trúc của lưới điện truyền tải có xét tổn thất công suất. Mô hình MILP đề xuất được xây dựng từ mô hình MINLP sử dụng kỹ thuật tuyến tính hóa từng đoạn và tuyến tính hóa chính xác tích của biến nhị phân và biến liên tục. Hàm mục tiêu của bài toán TNEP trong nghiên cứu này là tối thiểu hóa tổng chi phí của hệ thống điện, bao gồm vốn đầu tư xây dựng các đường dây, chi phí bảo dưỡng, chi phí tổn thất điện năng và chi phí sản xuất của các tổ máy phát điện. Các ràng buộc được xem xét bao gồm giới hạn ngân sách đầu tư, hệ phương trình trào lưu công suất, giới hạn truyền tải công suất trên các đường dây và giới hạn công suất phát của các tổ máy. Mô hình MILP đề xuất được đánh giá trên lưới điện 24 nút IEEE sử dụng bộ giải thương mại CPLEX với ngôn ngữ lập trình GAMS.

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1. Introduction

The increase in electricity demand consumed and the increasing integration of renewable energy sources significantly have an impact on operating and planning power systems. On the one hand, the power flow in the existing grids can be changed considerably by the load growth, which is likely to lead to some problems such as overload and instability. These problems may result in a violation of reliability criteria. On the other hand, renewable energy sources are usually installed in areas far from the load centres and are not readily connected to transmission networks. Therefore, the capacity of transmission power networks needs to be enhanced, contributing to the future power grids' flexibility and robustness to withstand numerous uncertain elements and disturbances.

Transmission network expansion planning (TNEP) has been researched extensively in electric power systems. TNEP aims to determine the time, location and number of transmission assets demanded in the future. Because of the increasing complexity of the network structure and the deregulation of the electricity industry, the TNEP is an intricate decision-making problem.

A variety of TNEP models have been put forward for several decades, in which mathematical programming and heuristic approaches are two significant solution methodologies. The former techniques usually ensure the optimal solution in most situations; however, they strictly require models to be optimized [1]. Nevertheless, the latter approaches have some advantages of their insensitivity to the model optimized. Regarding the disadvantages of heuristic methods, such methods do not guarantee an optimal solution and provide few clues in terms of the solution's quality.

Because the TNEP problem is highly complicated, the Direct Current Power Flow (DCPF) method has been effectively deployed for building TNEP models [2]. A disjunctive formulation based on mixed-integer linear programming (MILP) aiming at eliminating binary variables that pose to the nonlinear model was developed in [3]. In [4], the demand side bidding was integrated to create a model considering the load's behaviour. The paper [5] also developed a MILP-based method to deal with the multi-stage TNEP problem considering the N-1 security constraints. Short-circuit level constraints are approximately linear to be considered in the TNEP problem [6]. Authors [7] jointly considered the TNEP problem and the optimal location of thyristor-controlled series compensators. These planning problems are linearized using the general linearization technique and the disjunctive model. A bi-level optimization-based technique for planning transmission networks in electricity markets was suggested in [8]. A bi-level optimization-based transmission network expansion planning in a deregulated environment considering nonconvex cost curves was introduced in [9].

The TNEP model using DCPF does not take into account the active power losses that can shift the generating outputs, impacting the optimal investment strategy. Research into linearizing the quadratic loss term to be integrated into the TNEP model is proposed in [10].

The TNEP based on the Alternating Current power flow (ACPF) model is seldom presented in the literature. This is because the ACPF-based TNEP (ACTNEP) problem is nonlinear and nonconvex, making it challenging to resolve and achieve an optimal solution. In [11], the ACTNEP problem is solved using a combination of the standard branch-and-bound method and some of the standard nonlinear programming solvers supported by GAMS. Recently, an approach based on mixed-integer nonlinear programming (MINLP) has been introduced to plan transmission asset investment considering thyristor-controlled series compensators (TCSC) and superconducting fault current limiters (SFCL) [12].

This paper aims to deploy the TNEP model based on DCPF taking into account real power losses to find optimal transmission asset investment planning. This research has made significant contributions as below:

• A MILP-based formulation of the TNEP problem considering network power loss integrated into both constraints and objective functions is developed.

• The impact of network power loss and the operating cost of generating units on the optimal solution of the TNEP problem is analyzed.

The paper is structured into four sections. Section 2 presents the model and solution method of the TNEP problem using DCPF considering active power losses. Numerical results and discussions on the modified IEEE 24-bus transmission systems are given in Section 3, and the conclusions are drawn in Section 4.

2. Methodology

2.1. MINLP formulation of TNEP problem

The TNEP problem can be formulated as the following MINLP model:

$$\underset{x_{nk},P_{nk}^{\text{loss}},P_{nk},\delta_n,P_{\text{G}i}}{\text{minimize}} \sum_{\forall(n,k)\in\Omega^{\text{L+}}} c_{nk}x_{nk} + 0.02\sum_{\forall(n,k)\in\Omega^{\text{L+}}} c_{nk}x_{nk} + c_{tt}\sum_{\forall(n,k)\in\Omega^{\text{L}}} P_{nk}^{\text{loss}}\tau + 8760\sum_{i\in\Omega^{\text{G}}}\lambda_{\text{G}i}P_{\text{G}i} \quad (1)$$

where c_{nk} is the annualized investment cost of prospective transmission lines connected between nodes *n* and *k*; x_{nk} is the binary variable that is equal to 1 if prospective transmission line (*n*, *k*) is built and 0 otherwise; c_{tt} is the marginal cost of electricity; P_{nk}^{loss} is the real power loss in line (*n*, *k*); τ is the equivalent hours of energy loss; $\lambda_{\text{G}i}$ is the marginal cost of producing electricity by unit *i*; $P_{\text{G}i}$ is the power produced by generating unit *i*; $\Omega^{\text{L+}}$ is the set of prospective transmission lines, and Ω^{L} is the set of transmission lines, including existing lines and prospective ones.

The objective function (1) encompasses the four terms below: the annualized cost of building new transmission assets, the operating and maintenance (OM) cost, the cost of energy losses, and the operating cost of generating units, in which the OM cost is taken as 2% of the total investment cost.

This TNEP optimization model is subject to the following constraints:

$$\sum_{\forall(n,k)\in\Omega^{\text{L+}}} c_{nk}x_{nk} \le T \quad (2)$$

$$\sum_{i\in\Omega_n^{\text{G}}} P_{\text{G}i} - \sum_{j\in\Omega_n^{\text{D}}} P_{\text{D}j} = \sum_{k\in\Omega_n^{\text{L}}} P_{nk}; \quad \forall n\in N \quad (3)$$

$$P_{nk} = g_{nk}x_{nk}\left[1-\cos(\delta_n-\delta_k)\right] - b_{nk}x_{nk}\sin(\delta_n-\delta_k); \quad \forall(n,k)\in\Omega^{\text{L}} \quad (4)$$

$$-P_{nk}^{\max} \le P_{nk} \le P_{nk}^{\max}; \quad \forall(n,k)\in\Omega^{\text{L}} \quad (5)$$

$$-P_{nk}^{\max} \le P_{kn} \le P_{nk}^{\max}; \quad \forall(n,k)\in\Omega^{\text{L}} \quad (6)$$

$$P_{\text{G}i}^{\min} \le P_{\text{G}i} \le P_{\text{G}i}^{\max}; \quad \forall i\in\Omega^{\text{G}} \quad (7)$$

$$x_{nk} = 1; \quad \forall(n,k)\in\Omega^{\text{L}}\setminus\Omega^{\text{L+}} \quad (8)$$

$$x_{nk} = \{0,1\}; \quad \forall(n,k)\in\Omega^{\text{L+}} \quad (9)$$

where *T* is the annualized investment budget for constructing prospective transmission lines; $P_{\text{G}i}$ is the power produced by generator *i*; Ω_n^{G} is the set of generating units located in node *n*; $P_{\text{D}j}$ is the power consumed by demand *j*; Ω_n^{D} is the set of demand connected to node *n*; *N* is the set of all buses; δ_n is the voltage angle at node *n*; P_{nk} is the power flow through line (*n, k*); g_{nk} and b_{nk} are the real and imaginary parts of the series admittance of line (*n, k*), respectively; $P_{nk}^{\max}$ is the capacity of the transmission line (*n, k*); $P_{\text{G}i}^{\min}$ and $P_{\text{G}i}^{\max}$ are the lower and upper limits of power produced by generating unit *i*, respectively.

Constraints (2) guarantee that the cost of building new transmission lines is lower than the available budget. Constraints (3) impose the power balance at each bus of the power system. Equations (4) define power flows through transmission lines, bounded by the capacity limits according to constraints (5) and (6). Equations (4) are represented under the flat voltage assumption for normal operation. Limits for power generated by units are described by constraints (7). Constraints (8) and (9) define binary variables.

The above optimization model (1)-(9) is the MINLP problem due to constraints (3)-(6). Subsection 2.2 presents different linearization methods to convert this MINLP model to the MILP form.

2.2. MILP formulation of TNEP problem

2.2.1. Linearization of network losses

In this section, the expressions presented below apply to every transmission line; therefore, the indication $\forall(n,k)\in\Omega^{\mathrm{L}}$ will be explicitly omitted.

The real power flows in the line (*n*, *k*) determined at bus *n* and *k*, respectively, are given by

$$P_{nk}=g_{nk}x_{nk}\left[1-\cos\left(\delta_n-\delta_k\right)\right]-b_{nk}x_{nk}\sin\left(\delta_n-\delta_k\right) \tag{10}$$

$$P_{kn}=g_{nk}x_{nk}\left[1-\cos\left(\delta_k-\delta_n\right)\right]+b_{nk}x_{nk}\sin\left(\delta_k-\delta_n\right) \tag{11}$$

The real power loss in the line (*n*, *k*) can be attained as follows:

$$P_{nk}^{\mathrm{loss}}\left(\delta_n,\delta_k,x_{nk}\right)=P_{nk}+P_{kn}=2g_{nk}x_{nk}\left[1-\cos\left(\delta_n-\delta_k\right)\right]\cong g_{nk}x_{nk}\left(\delta_n-\delta_k\right)^2 \tag{12}$$

The losses function (12) can be expressed using piecewise linear approximation [13]:

$$P_{nk}^{\mathrm{loss}}\left(\delta_n,\delta_k,x_{nk}\right)=g_{nk}x_{nk}\sum_{l=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right) \tag{13}$$

$$\left|\delta_n-\delta_k\right|=\sum_{\ell=1}^{L}\delta_{nk}(\ell) \tag{14}$$

$$\delta_{nk}\left(\ell\right)\geq 0;\quad \ell=1,...,L \tag{15}$$

$$\delta_{nk}\left(\ell\right)\leq\Delta\delta+\left(1-x_{nk}\right)M_{nk};\quad \ell=1,...,L \tag{16}$$

where $\alpha_{nk}\left(\ell\right)$ and $\delta_{nk}\left(\ell\right)$ define the slope and value of the *l*th block of voltage angle of the line (*n*, *k*); M_{nk} is the sufficiently big non-negative constant; $\Delta\delta$ is the upper limit of the angle blocks; *L* is the number of blocks of the piecewise linearization of power losses.

Constraints (14) denote that the absolute value of the angle difference between two buses equals the sum of the discretization block's values. Constraints (15) and (16) impose the lower and upper bounds of each angle block contributed to the total angle difference for the line (*n*, *k*).

The constraints (17)–(20) are needed to impose the adjacent blocks of the angles.

$$\mathrm{w}_{nk}\left(\ell\right)\Delta\delta\leq\delta_{nk}\left(\ell\right);\quad \ell=1,...,L-1 \tag{17}$$

$$\delta_{nk}\left(\ell\right)\leq \mathrm{w}_{nk}\left(\ell-1\right)\Delta\delta;\quad \ell=2,...,L \tag{18}$$

$$\mathrm{w}_{nk}\left(\ell-1\right)\geq \mathrm{w}_{nk}\left(\ell\right);\quad \ell=2,...,L-1 \tag{19}$$

$$\mathrm{w}_{nk}\left(\ell\right)\in\left\{0;1\right\};\quad \ell=1,...,L-1 \tag{20}$$

where $\mathrm{w}_{nk}\left(\ell\right)$ is binary variables that are equal to 1 if the angle value of the ℓth linearization block equals $\Delta\delta$.

Moreover, the slopes of the angle blocks of line flow $\alpha_{nk}\left(\ell\right)$ for all transmission lines can be given by equation (21):

$$\alpha_{nk}\left(\ell\right)=\left(2\ell-1\right)\Delta\delta_{nk}^{\max} \tag{21}$$

It is emphasized that the number of linear segments will radically affect the optimal problem solution's accuracy. Moreover, this linear technique is not influenced by selecting the reference bus. A linear expression of the absolute value in (14) is needed, which is obtained by employing the following substitutions:

$$\left|\delta_n-\delta_k\right|=\delta_{nk}^{+}+\delta_{nk}^{-};\quad \delta_n-\delta_k=\delta_{nk}^{+}-\delta_{nk}^{-};\quad \delta_{nk}^{-}\geq 0;\quad \delta_{nk}^{+}\geq 0 \tag{22}$$

The real power flow in the line (n, k) computed at bus n and k can be recast as follows, respectively:

$$\begin{aligned}P_{nk}&=\frac{1}{2}P_{nk}^{\text{loss}}-b_{nk}x_{nk}\sin\left(\delta_n-\delta_k\right)=\frac{1}{2}g_{nk}x_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)-b_{nk}x_{nk}\left(\delta_n-\delta_k\right)\\&=x_{nk}\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)-b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\end{aligned} \tag{23}$$

$$\begin{aligned}P_{kn}&=\frac{1}{2}P_{nk}^{\text{loss}}+b_{nk}x_{nk}\sin\left(\delta_n-\delta_k\right)=\frac{1}{2}g_{nk}x_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)+b_{nk}x_{nk}\left(\delta_n-\delta_k\right)\\&=x_{nk}\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)+b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\end{aligned} \tag{24}$$

Mathematical expressions (23) and (24) are nonlinear because of the product of binary and continuous variables.

2.2.2. Linearization of products of binary and continuous variables

The nonlinear constraints (23)-(24) can be replaced with the following sets of exact equivalent mixed-integer linear constraints:

$$-x_{nk}P_{nk}^{\max}\leq P_{nk}\leq x_{nk}P_{nk}^{\max} \tag{25}$$

$$P_{nk}-\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)-b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\geq-\left(1-x_{nk}\right)M_{nk} \tag{26}$$

$$P_{nk}-\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)-b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\leq\left(1-x_{nk}\right)M_{nk} \tag{27}$$

$$-x_{nk}P_{nk}^{\max}\leq P_{kn}\leq x_{nk}P_{nk}^{\max} \tag{28}$$

$$P_{kn}-\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)+b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\geq-\left(1-x_{nk}\right)M_{nk} \tag{29}$$

$$P_{kn}-\left[\frac{1}{2}g_{nk}\sum_{\ell=1}^{L}\alpha_{nk}\left(\ell\right)\delta_{nk}\left(\ell\right)+b_{nk}\left(\delta_{nk}^{+}-\delta_{nk}^{-}\right)\right]\leq\left(1-x_{nk}\right)M_{nk} \tag{30}$$

where M_{nk} is a large enough positive constant.

2.2.3. MILP model formulation

The problem of transmission network expansion planning can be reformulated as the MILP problem using the linearization procedure described above:

$$\underset{\substack{x_{nk},P_{nk}^{\text{loss}},P_{nk},\delta_n,\\ P_{Gi},\delta_{nk}(\ell),\delta_{nk}^{+},\delta_{nk}^{-}}}{\text{minimize}}\ \sum_{\forall(n,k)\in\Omega^{L+}}c_{nk}x_{nk}+0.02\sum_{\forall(n,k)\in\Omega^{L+}}c_{nk}x_{nk}+c_{tt}\sum_{\forall(n,k)\in\Omega^{L}}P_{nk}^{\text{loss}}\tau+8760\sum_{i\in\Omega^{G}}\lambda_{Gi}P_{Gi} \tag{31}$$

$$\text{Constraints }(2)-(3);(7)-(9);(17)-(20);(25)-(27);(29)-(31) \tag{32}$$

$$\delta_{nk}^{+}+\delta_{nk}^{-}=\sum_{\ell=1}^{L}\delta_{nk}(\ell);\qquad\forall(n,k)\in\Omega^{L} \tag{33}$$

$$\delta_n=0\quad n:\text{reference bus} \tag{34}$$

$$\delta_{nk}^{-} \geq 0; \delta_{nk}^{+} \geq 0; \quad \forall (n,k) \in \Omega^{L} \tag{35}$$

$$\delta_{nk}(\ell) \geq 0; \quad \forall (n,k) \in \Omega^{L}; \ \ell = 1,...,L \tag{36}$$

$$\delta_{nk}(\ell) \leq \Delta\delta + (1 - x_{nk}) M_{nk}; \ \forall (n,k) \in \Omega^{L}; \ell = 1,...,L \tag{37}$$

3. Result and discussion

This section presents the application of the developed model to the modified IEEE 24-bus network [14]. The optimization model is solved using CPLEX called from the GAMS environment [15]. The case study is carried out on an AMD Ryzen 5 5600G, 3.9 GHz-based processor with 32GB RAM.

A modified IEEE 24-bus system is depicted in Figure 1. Initially, the number of existing lines per corridor is 1. The total power of loads for this network is 2850 MW. The maximum number of built and candidate lines in each transmission corridor is set to 3. The equivalent hours of energy loss are assumed to be 3500 h. The marginal cost of energy loss is taken as 25 \$/MWh. Moreover, to make investment decisions, the flow limits of each transmission line are decreased to one-third of those shown in [14].

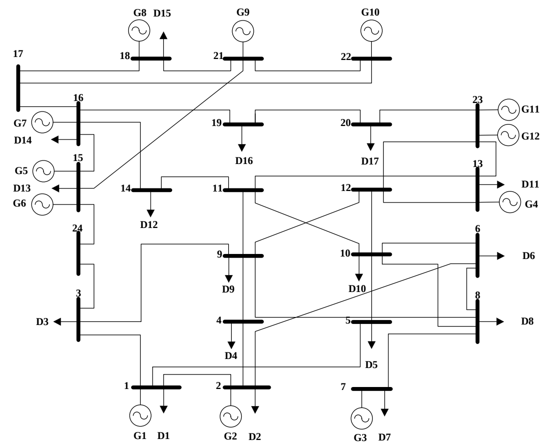

Figure 1. *IEEE 24-bus system*

3.1. Selection of the number of linear segments

The number of linear segments significantly affects the calculation time and the model accuracy. The larger the number of linear segments, the closer the network power loss is to the quadratic expression. Table 1 describes the effects of the number of linear segments on the solution result and time. The following observations are drawn:

- When the number of linear segments is less than 7, the problem is infeasible with the given set of prospective lines.
- As the number of linear blocks is from 7 to 9, the value of the objective function decreases.
- When the number of linear blocks is greater than or equal to 9, the results, including the objectives, the investment cost and the total power loss, maintain unchanged.
- The solution time increases as the number of linear blocks grows.

Therefore, for IEEE 24-bus system, the number of linear blocks is selected as 9.

Table 1. *The influence of the number of linear blocks*

Linear blocks	Objective (M\$)	Investment cost (M\$)	Total power loss (MW)	Calculation time (s)
1÷6		Infeasible		
7	592.36	278.32	28.93	3.477
8	588.24	278.32	30.02	11.625
9	**548.21**	**255.35**	**33.3**	**14.166**
10	548.21	255.35	33.3	15.055
11	548.21	255.35	33.3	16.210
12	548.21	255.35	33.3	19.310

3.2. The effects of network power loss

The expanded IEEE 24-node system in terms of the number of transmission lines to be built and the total investment cost, when considering and taking no account of network power loss, respectively, is shown in Table 2. The branch power flow and power loss on each corridor of these two cases are presented in Table 3.

Calculation results show that considering and neglecting power loss significantly affects the decision of line investment and the power flowing through branches. For the case of considering power loss, 8 new lines need to be built, including one transmission line for corridors 6–10, 7–8, 10–12, 14–16, 15–21, 15–24 and 16–17. The total investment cost is 255.3 M$. The grid power loss is 33.30 MW, accounting for 1.15% of the total load. Meanwhile, in the absence of power loss, the number of new lines to be constructed is 8, including two lines linking nodes 7–8 and one edge for corridors 6–10, 10–11, 11–13, 14–16, 16–17 and 20–23. The investment cost is 203.1 M$.

Table 2. *The impact of network loss on the optimal solution*

TNEP model	Expansion plan	Investment cost (M$)
With loss	3-24; 6–10; 7–8; 10–12 14–16; 15–21; 15–24; 16–17	255.3
Without loss	6–10; 7–8 (× 2); 10–11 11–13; 14–16; 16–17; 20–23	203.1

Table 3. *Branch power flow and power loss*

Corridor	Loss P_{nk} (MW)	Loss P_{nk}^{loss} (MW)	No loss P_{nk} (MW)	Corridor	Loss P_{nk} (MW)	Loss P_{nk}^{loss} (MW)	No loss P_{nk} (MW)
1-2	12.19	0.04	-6.34	11-13	-122.15	0.95	-148.48
1-3	-19.70	0.24	16.67	11-14	-119.08	0.79	-36.72
1-5	51.50	0.63	33.67	12-13	-158.25	1.59	-116.53
2-4	37.55	0.49	33.46	12-23	-150.02	2.90	-144.46
2-6	29.60	0.47	15.21	13-23	-81.26	0.76	-97.21
3-9	44.22	0.64	-30.00	14-16	-156.94	1.29	-115.36
3-24	-122.08	0.35	-133.33	15-16	97.14	0.22	-75.39
4-9	-36.94	0.41	-40.54	15-21	-148.69	1.44	-166.67
5-10	-20.13	0.11	-37.33	15-24	123.47	1.04	133.33
6-8	1.95	0.13	-10.69	16-17	-154.19	0.83	-132.72
6-10	-54.41	0.44	-55.05	16-19	143.86	0.64	14.33
7-8	58.33	0.59	58.33	17-18	-166.10	0.57	-128.14
8-9	-31.34	0.46	-2.13	17-22	-143.95	2.90	-137.30
8-10	-22.35	0.24	-4.56	18-21	-100.32	0.38	-61.14
9-11	-110.76	0.29	-114.63	19-20	-37.78	0.09	-166.67
9-12	-89.81	0.19	-133.04	20-23	-165.87	0.80	-147.33
10-11	-129.79	0.39	-109.53	21-22	-151.10	2.05	-162.70
10-12	-108.86	0.28	-127.94				

3.3. The influence of the operating cost of generators

In this subsection, the expansion plan, comprising new lines to be erected, the total investment cost, and the power output of generators, is compared when considering and neglecting the inclusion of the operating cost of generators in the objective function. Illustrating the calculation results for these cases is presented in Table 4 and Table 5. It is noted that the network power loss is taken into account for both cases.

As can be observed from Table 4 and Table 5, including and excluding the operating cost of generators can considerably impact the optimal solution. Particularly, when integrating the generator's operating cost into the objective function, the number of new transmission lines linking buses 7 and 8 to be constructed is 1. Meanwhile, this corridor needs building two new lines when the operating cost of generating units is not integrated into the objective function. In addition, the total investment cost for both cases is 255.3 M$ and 232.3 M$, respectively.

Table 4. *The effect of the operating cost of generating units on the expansion plan*

TNEP model	Expansion plan	Investment cost (M$)
With the operating cost of generators	3-24; 6–10; 7–8; 10–12; 14–16 15–21; 15–24; 16–17	255.3
Without the operating cost of generators	6–10; 7–8 (× 2); 10–11; 11–13 12–23; 16–17; 20–23	232.3

Table 5. *Optimal generating output (MW) of units*

Generator	Inclusion of the operating cost of generators	Exclusion of the operating cost of generators
1	152.00	152.00
2	152.00	152.00
7	**241.67**	**286.63**
13	**466.68**	**591.00**
15	60.00	60.00
15	155.00	155.00
16	155.00	155.00
18	**399.35**	**330.72**
21	**400.00**	**100.00**
22	300.00	300.00
23	**261.61**	**310.00**
23	**140.00**	**286.07**
Total power output	**2888.31**	**2878.42**

4. Conclusion

This paper proposes a model based on mixed-integer linear programming to optimize the transmission grid expansion plan considering the network power loss. The objective function is to minimize the total cost of the power system, including the investment cost of building new lines, maintenance cost, energy loss cost and operating cost of generating units. The model deploys the piecewise linearization technique for the network power loss and precise linearization of the product of binary and continuous variables. The selection of the number of linear blocks is rigorously analyzed. The optimal results reveal that the network power loss and the operating cost of generators have a significant influence on the expansion plan, such as the number of new lines to be erected, the total investment cost, the unit's power output and the branch power flow.

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