ON THE FIXED POINT OF FREQUENCY RESPONSE OF SINGLE DEGREE-OF-FREEDOM BASE EXCITED ISOLATION WITH PIEZOELECTRIC ENERGY HARVESTING

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ABSTRACT

Vibration isolation is a fundamental device to protect machines or vehicles from unwanted vibration. Adding the energy harvesting capability to the vibration isolation can extract the waste heat energy. This paper considers the single-degree-of-freedom (SDOF) base excited isolation with piezoelectric energy harvesting. The purpose of this paper is to find how the energy harvesting circuit affects the fixed point of the frequency response. The fixed point is an important point in design the vibration isolation. The analytical frequency response method is used. The motion equation is written in the dimensionless form. The frequency response analysis of the system reveals the fixed point in the frequency response plot. The position of the fixed point does not depend on the circuit resistance. Two parameters of the piezoelectric harvesting circuit including the dimensionless resistance and the dimensionless force factor can be changed in isolation design. The design strategy is to make the fixed point and the peak point coinciding. Based on the strategy, the optimal condition of the isolation is expressed by a relation between the dimensionless resistance and the dimensionless force factor.

VỀ ĐIỂM CỐ ĐỊNH TRÊN ĐÁP ỨNG TẦN SỐ CỦA BỘ CÁCH LY DAO ĐỘNG KÍCH ĐỘNG NỀN MỘT BẬC TỰ DO KÈM THEO THU HOẠCH NĂNG LƯỢNG ÁP ĐIỆN

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TỪ KHÓA

Cách ly dao động
Thu hoạch năng lượng
Điểm cố định
Đáp ứng tần số
Phân tích phi thứ nguyên

TÓM TẮT

Bộ cách ly dao động là một thiết bị cơ bản để bảo vệ máy móc và phương tiện khỏi các dao động không mong muốn. Bổ sung khả năng thu hoạch năng lượng cho bộ cách ly có thể trích xuất được các năng lượng nhiệt hao phí. Bài báo này quan tâm tới bộ cách ly dao động kích động nền một bậc tự do kèm theo thu hoạch năng lượng áp điện. Mục đích của bài báo là tìm hiểu xem mạch thu hoạch năng lượng ảnh hưởng như thế nào đến điểm cố định của đáp ứng tần số. Điểm cố định là điểm quan trọng trong thiết kế bộ cách ly. Phương pháp đáp ứng tần số được sử dụng. Phương trình chuyển động được viết dưới dạng phi thứ nguyên. Phân tích đáp ứng tần số của hệ cho thấy sự xuất hiện của điểm cố định trên đồ thị đáp ứng tần số. Vị trí của điểm cố định không phụ thuộc vào điện trở mạch. Hai tham số của mạch thu hoạch áp điện bao gồm điện trở và hệ số lực không thứ nguyên có thể được thay đổi trong thiết kế bộ cách ly. Chiến lược thiết kế là làm cho điểm cố định và điểm cực đại trùng nhau. Dựa trên chiến lược này, điều kiện tối ưu của bộ cách ly được biểu thị bằng mối quan hệ giữa điện trở và hệ số lực không thứ nguyên.

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1. Introduction

The base of the system, in some applications, is subjected to an unwanted vibration. For example, the foundation of a machine is subjected to earthquake; a delicate instrument is experienced vibration transmitted from the excessive base motion. A suitably designed isolation system is required to prevent the damage, failure and improve the performance. The conventional vibration isolation dissipates energy as waste heat in the damper. Vibration energy harvesting is the field studying the conversion of vibration energy into useful electrical energy. Piezoelectric is a well-known transducer mechanism in vibration energy harvesting. The recent review [1] showed the significant interest of researchers and engineers in the dual function of vibration control and energy harvesting simultaneously.

Energy harvesting by piezoelectric vibration harvester has attracted a lot of attentions in recent years [2] – [6]. In the conventional vibration isolation with viscous damping, there is a fixed point in the frequency response plot, which is important to the isolation design. Above the frequency of fixed point, the addition of damping increases the vibration transmissibility and conversely [7]. Therefore, the adaptive damping may be considered to achieve the best performance.

When the piezoelectric transducer is added, there is no study to reveal if the fixed point exists. In this paper, we show the position of the fixed point and how to use this point to design the isolation.

2. Modeling and analysis

The base excited piezoelectric harvester is modeled in Figure 1, which consists of a spring-mass oscillator, a piezoelectric transducer, and a load resistor.

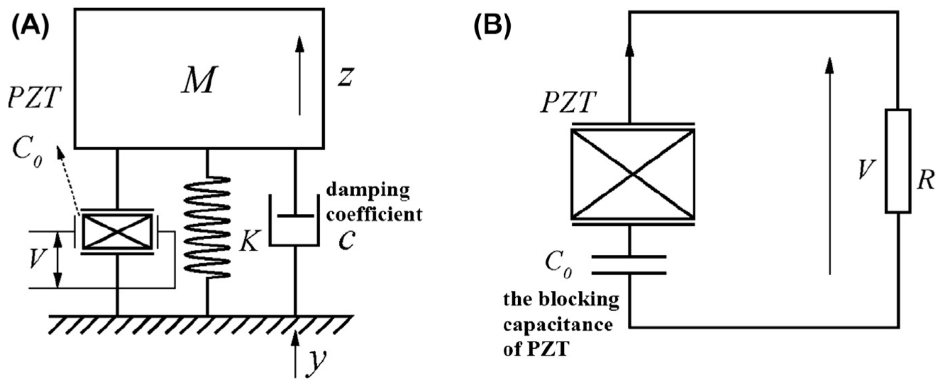

Figure 1. *Model of piezoelectric harvester (A) the oscillator (B) the energy harvesting circuit*

The mechanical motion equation is given by:

$$M\ddot{z} + c\dot{z} + Kz = -M\ddot{y} - \alpha V \tag{1}$$

while the electrical equation is governed by:

$$\frac{V}{R} = \alpha\dot{z} - C_0\dot{V} \tag{2}$$

where M is the mass, c and K are the short circuit mechanical damping and stiffness, respectively, y is the displacement of the base, z is the relative displacement of the mass with respect to the base, α is the force factor, R is the circuit resistance, V is the voltage across R, and C_0 is the piezoelectric capacitance.

Assume the harmonic base excitation as:

$$y = Ye^{i\omega t} \tag{3}$$

where i is the imaginary unit and ω is circular frequency. As the system is linear, the output voltage V and the relative displacement z are harmonic too:

$$\begin{aligned} z &= Ze^{i\omega t} \\ V &= V_0 e^{i\omega t} \end{aligned} \tag{4}$$

Substituting (3) and (4) into (1) and (2) gives:

$$\begin{cases} \left(-m\omega^2 + ic\omega + k\right)Z = m\omega^2 Y - \alpha V_0 \\ \dfrac{V_0}{R} = i\alpha\omega Z - iC_0\omega V_0 \end{cases} \tag{5}$$

Solving (5) we have the displacement transmissibility expressed by [11]:

$$T = \frac{Z+Y}{Y} = \frac{\left(\alpha^2 + kC_0\right)i\omega R + ci\omega + k - \omega^2 cC_0 R}{\left(\alpha^2 - m\omega^2 C_0 + kC_0\right)i\omega R + ci\omega - m\omega^2 + k - \omega^2 cC_0 R} \tag{6}$$

The amplitude of T is determined by:

$$|T| = \sqrt{\frac{\left(\left(\alpha^2 + kC_0\right)R + c\right)^2 \omega^2 + \left(k - \omega^2 cC_0 R\right)^2}{\left(\left(\alpha^2 - m\omega^2 C_0 + kC_0\right)R + c\right)^2 \omega^2 + \left(k - m\omega^2 - \omega^2 cC_0 R\right)^2}} \tag{7}$$

Denote the following non-dimensional terms:

$$\omega_N = \frac{\omega}{\omega_0} = \frac{\omega}{\sqrt{K/M}}; R_N = RC_0\omega_0; \alpha_N = \frac{\alpha}{\sqrt{KC_0}}, \zeta = \frac{c}{2M\omega_0} \tag{8}$$

where ω_N is the normalized forcing frequency, ω_0 is the natural frequency, ζ is the damping ratio, α_N is the non-dimensional force factor and R_N is the non-dimensional resistance.

The amplitude of the displacement transmissibility (7) is written in the dimensionless form:

$$|T| = \sqrt{\frac{\left(\left(\alpha_N^2 + 1\right)R_N + 2\zeta\right)^2 \omega_N^2 + \left(1 - \omega_N^2 2\zeta R_N\right)^2}{\left(\left(\alpha_N^2 - \omega_N^2 + 1\right)R_N + 2\zeta\right)^2 \omega_N^2 + \left(1 - \omega_N^2 - \omega_N^2 2\zeta R_N\right)^2}} \tag{9}$$

To find the position of the fixed point, let ζ be zero in (9) we have:

$$|T|_{FP} = \sqrt{\frac{\left(\alpha_N^2 + 1\right)^2 R_N^2 \omega_N^2 + 1}{\left(\alpha_N^2 - \omega_N^2 + 1\right)^2 R_N^2 \omega_N^2 + \left(1 - \omega_N^2\right)^2}} \tag{10}$$

The condition of fixed point:

$$\frac{\partial |T|_{FP}^2}{\partial R_N^2} = 0 \tag{11}$$

will result in the frequency of fixed point. The position of the fixed point is a good result to design the isolator. Based on the very well-known idea of Den Hartog [8], the optimal strategy is to make the response curve as flat as possible at fixed points [7]. This can be explained as follows. If the curve has a positive slope at the fixed point, a higher peak will appear on the right of the fixed point. Conversely, a negative slope at the fixed point creates a higher peak on the left. Therefore the good frequency response should be the one in which the fixed point coincides with the peak point.

The peak point of (10) is found by the condition:

$$\frac{\partial |T|_{FP}^2}{\partial \omega_N^2} = 0 \tag{12}$$

Combining (11) and (12) we obtain a relation between R_N and α_N in design

3. Results and Discussion

In Figures 2, 3, 4, and 5, the frequency response (9) is plotted versus ω_N for various values of ζ, α_N and R_N.

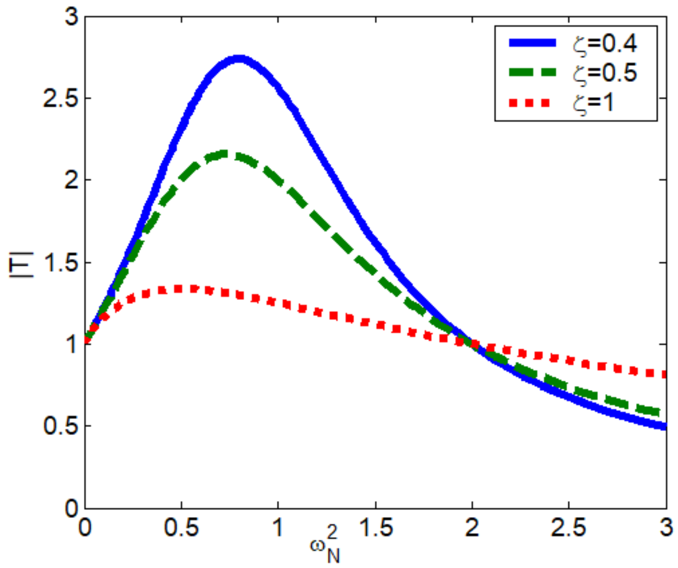

Figure 2. *Frequency response for R_N=0*

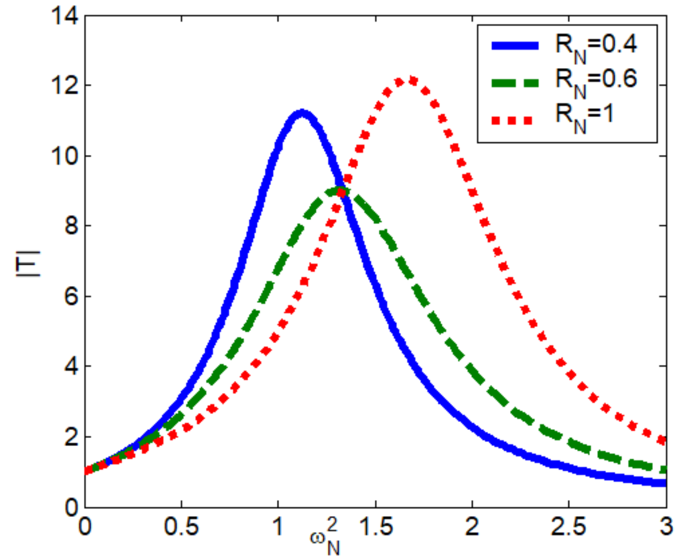

Figure 3. *Frequency response for ζ=0, α_N=1*

As seen in Figure 2, when R_N=0 (short circuit, no harvesting), there is a fixed point regardless of the value of damping ratio ζ. This is the conventional behavior of vibration isolation with viscous damping. In Figures 3 and 4, when ζ=0 (no mechanical damping), there is a fixed point regardless of the value of dimensionless resistance R_N. When α_N changes, this fixed point moves and the peak of the frequency response also moves. In Figure 5, when both ζ and R_N are non-zeros, the fixed point does not exist. However, for small mechanical damping ration ζ, the nearly fixed point can be observed.

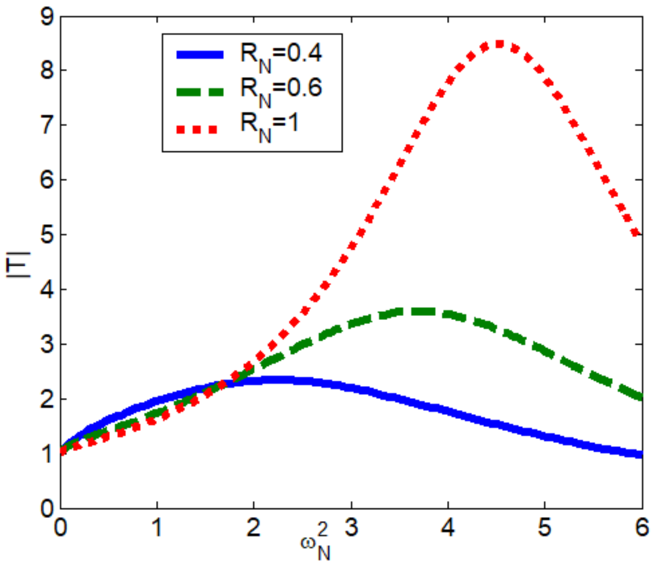

Figure 4. *Frequency response for ζ=0, α_N=2*

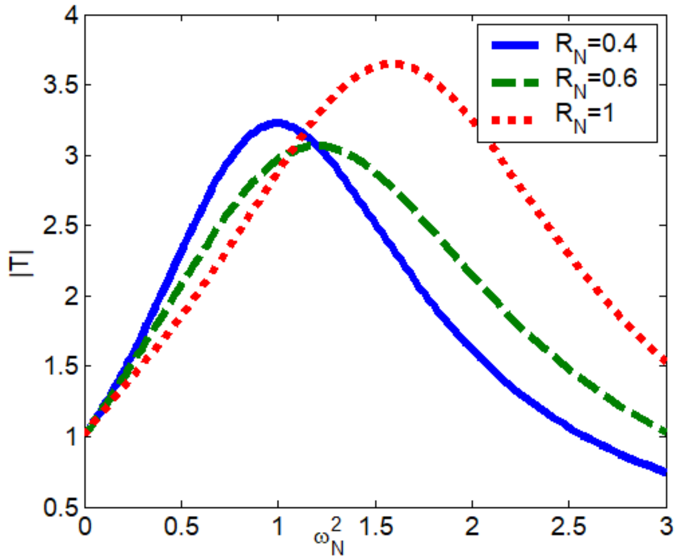

Figure 5. *Frequency response for ζ=0.5, α_N=1*

Next, we find the position of the fixed point by (11). Some manipulations from (11) yields the frequency of fixed point:

$$\omega_{Nf}^2 = \frac{2\left(\alpha_N^2 + 1\right)}{\alpha_N^2 + 2} \tag{13}$$

where ω_{Nf} is the normalized frequency of the fixed point. The displacement transmissibility at the fixed point is also found by substituting (13) into (10):

$$|T|_f = \frac{1}{\left|1 - \omega_{Nf}^2\right|} = \frac{\alpha_N^2 + 2}{\alpha_N^2} \tag{14}$$

Some conclusions can be drawn from (13) and (14). First, if the force factor is large (α_N tends to infinity), the normalized frequency of fixed point (13) tends to $\sqrt{2}$, which is the conventional result [7], [11]. Second, the displacement transmissibility at the fixed point (14) is larger than 1. This implies the performance of the isolation with energy harvesting is worse than the conventional one. This may be the price paid to harvest energy.

The condition (12) of the peak point leads to the following equation of ω_N:

$$\begin{aligned}&2R_N^4\left(\alpha_N^2+1\right)^2\omega_N^6+\left(-2R_N^2\left(\alpha_N^2+1\right)^3+\alpha_N^4+4+2\alpha_N^2\right)R_N^2\omega_N^4\\&+2\left(1-2R_N^2\left(\alpha_N^2+1\right)\right)\omega_N^2-2=0\end{aligned} \tag{15}$$

Substituting the frequency of fixed point (13) into (15) we obtain a relation between R_N and α_N. After some manipulations we have:

$$R_{N\_opt}=\frac{1}{\alpha_N^2+1}\sqrt{\frac{\alpha_N^2+2}{2}} \tag{16}$$

where R_{N_opt} is the optimal of the dimensionless resistance.

4. Conclusion

This paper found the fixed point in the frequency response plot of the displacement transmissibility of a SDOF base excited isolation with piezoelectric energy harvesting. The position of the fixed point does not move with the dimensionless resistance. The displacement transmissibility at the fixed point is larger than unity and this may be the price paid for the energy harvesting. To design the isolator, the idea of Den Hartog is used to match the fixed point with the peak. The result is a formula to calculate the dimensionless resistance based on the dimensionless force factor.

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REFERENCES

[1] Q. Cai and S. Zhu, “The nexus between vibration-based energy harvesting and structural vibration control: A comprehensive review,” *Renew. Sustain. Energy Rev.*, vol. 155, 2022, Art. no. 111920.

[2] H. Xiao, X. Wang, and S. John, “A dimensionless analysis of a 2DOF piezoelectric vibration energy harvester,” *Mech. Syst. Signal Process*, vol. 58-59, pp. 355-375, 2015.

[3] H. Xiao, X. Wang, and S. John, “A multi degree of freedom piezoelectric vibration energy harvester with piezoelectric elements inserted between two nearby oscillators,” *Mech. Syst. Signal Process,* vol. 68-69, pp. 138-154, February 2016.

[4] H. Xiao and X. Wang, “A review of piezoelectric vibration energy harvesting techniques,” *Int. Rev. Mech. Eng.*, vol. 8, no. 3, pp. 609-620, 2014.

[5] X. Wang and L. W. “Lin Dimensionless optimization of piezoelectric vibration energy harvesters with different interface circuits,” *Smart Mater Struct.* vol. 22, pp. 1-20, 2013.

[6] X. Wang and H. Xiao, “Dimensionless analysis and optimization of piezoelectric vibration energy harvester,” *Int. Rev. Mech. Eng*., vol. 7, no. 4, pp. 607-624, 2013.

[7] S. S. Rao, *Mechanical vibrations*, Pearson Education, Harlow, UK, 2018.

[8] J. Ormondroyd and J. P. D. Hartog, “The theory of the dynamic vibration absorber,” *Transactions of ASME*, vol. 50, pp. 9-22, 1928.

[9] A. Preumont, *Vibration Control of Active Structures*, Springer-Verlag Berlin Heidelberg, 2011.