ADAPTIVE NEURAL NETWORK-BASED TERMINAL SLIDING MODE CONTROL FOR A QUADROTOR UNMANNED AERIAL VEHICLE WITH DISTURBANCES

Duong Quynh Nga, Dang Ngoc Trung*
TNU - University of Technology

ARTICLE INFO

Received: 11/4/2025
Revised: 09/5/2025
Published: 09/5/2025

KEYWORDS

Quadrotor
Unmanned aerial vehicles
Adaptive control
Terminal sliding mode control
Neural network

ABSTRACT

This study presents an adaptive neural network-based terminal sliding mode control to ensure the trajectory tracking and stability of the Quadrotor unmanned aerial vehicle under unknown external disturbances. It is well known that the problem of Quadrotor tracking control tackles key challenges, including nonlinearity, coupling, model uncertainties, and external disturbances. To overcome these problems, an adaptive control scheme based on radial basic function neural network and terminal sliding mode control theory is proposed. In particular, the radial basic function neural network is employed to estimate the model uncertainties and external disturbances and a terminal sliding mode controller is used to achieve a good trajectory tracking performance and guarantee the stability of Quadrotor system. The stability of the closed-loop Quadrotor aerial vehicle is rigorously proven using the Lyapunov stability theory. Co-simulation using MATLAB/Simulink is provided to confirm the effectiveness of the proposed controller.

ĐIỀU KHIỂN TRƯỢT DỰA TRÊN MẠNG NƠ-RON THÍCH NGHI CHO THIẾT BỊ BAY KHÔNG NGƯỜI LÁI BỐN CÁNH QUẠT DƯỚI TÁC ĐỘNG CỦA NHIỄU

Dương Quỳnh Nga, Đặng Ngọc Trung*
Trường Đại học Kỹ thuật Công nghiệp – ĐH Thái Nguyên

THÔNG TIN BÀI BÁO

Ngày nhận bài: 11/4/2025
Ngày hoàn thiện: 09/5/2025
Ngày đăng: 09/5/2025

TỪ KHÓA

Thiết bị bay không người lái bốn cánh
Thiết bị bay không người lái
Điều khiển thích nghi
Điều khiển trượt terminal
Mạng nơ-ron

TÓM TẮT

Nghiên cứu này đề xuất một chiến lược điều khiển trượt kết hợp với mạng nơ-ron thích nghi nhằm đảm bảo bám quỹ đạo chính xác và duy trì ổn định cho thiết bị bay không người lái bốn cánh (Quadrotor) trong điều kiện có nhiễu bên ngoài không xác định. Như đã biết, bài toán điều khiển bám quỹ đạo cho Quadrotor đối mặt với nhiều thách thức lớn do tính phi tuyến nội tại, sự liên kết chặt chẽ giữa các trục (coupling), bất định mô hình và ảnh hưởng của nhiễu từ môi trường. Để khắc phục các vấn đề này, một cấu trúc điều khiển thích nghi được phát triển bằng cách tích hợp mạng nơ-ron xuyên tâm với bộ điều khiển trượt . Trong cấu trúc này, mạng nơ-ron xuyên tâm được sử dụng để xấp xỉ các bất định mô hình và nhiễu bên ngoài theo thời gian thực, trong khi bộ điều khiển trượt có vai trò đảm bảo hiệu suất bám quỹ đạo cao và hội tụ trong thời gian hữu hạn, đồng thời tăng cường khả năng chống nhiễu cho hệ thống. Tính ổn định của hệ thống vòng kín được chứng minh một cách chặt chẽ dựa trên lý thuyết ổn định Lyapunov. Cuối cùng, mô phỏng được thực hiện trên nền tảng MATLAB/Simulink được đưa ra để chứng minh hiệu quả và độ tin cậy của bộ điều khiển được đề xuất.

DOI: https://doi.org/10.34238/tnu-jst.12556

* Corresponding author. *Email: trungcsktd@tnut.edu.vn*

1. Introduction

In recent years, Quadrotor unmanned aerial vehicles (UAVs) have attracted much attention due to a variety of applications, such as logistics [1], search and rescue [2], mapping [3], photography [4], and so on. For the Quadrotor UAVs, the problem of trajectory tracking control is always one of the key issues. A numerous control approaches have been carried out on quadrotor platforms, such as PID control [5], LQR control [6], Backstepping control [7], and Model predictive control (MPC) [8]. However, the control design for the Quadrotor system usually requires tackling several challenges, including nonlinear dynamics, strong coupling, model uncertainties, and external disturbances [9] - [11]. Therefore, these model-based control approaches in [5] - [8] usually deliver low performance in real-world applications. The control problem for the quadrotor system is to design a robust controller that achieves trajectory tracking under the influence of dynamic uncertainties and external disturbances.

Sliding Mode Control (SMC) is a nonlinear control method, which is well-known as an effective method against uncertainties and disturbances. Due to its simplicity and robustness, the method has been studied extensively for over 50 years and has received many applications [12], [13]. The core idea of SMC lies in two fundamental problems: constructing the sliding surface and formulating a control law that ensures system stability, typically based on Lyapunov stability theory. A conventional SMC with a linear sliding surface is often designed to ensure the asymptotic stability of the system [14]. An advanced version, known as Terminal Sliding Mode Control (TSMC), has been developed to further enhance performance by ensuring that the tracking error converges to zero in a finite time, depending on the initial conditions [15]. However, there are several typical SMC design methods, for detailed discussions on them and key challenging issues such as chattering or sensitivity to unmodeled dynamics.

The limitations of the SMC methods are that the parameters of the dynamic system need to be known in advance so that it is more difficult to be satisfied in practical application. As far as we know, Radial Basis Function Neural Networks (RBFNNs) is a powerful technique for their approximation capabilities [16] - [18]. With a universal function approximation property, RBFNNs are usually utilized to approximate unknown nonlinear dynamics and uncertainties without requiring accurate mathematical models. They act as adaptive estimators or compensators that help cancel out the unknown parts of the dynamics. As a result, RBFNNs are commonly integrated into nonlinear control methods such as SMC to ensure system stability and desired performance despite the presence of uncertainties.

Inspired by these discussions, this paper presents a TSMC-based RBFNN framework for Quadrotor UAVs subject to external disturbances and model uncertainties. In particular, an RBFNN is employed to estimate the model uncertainties and external disturbances. Then, a TSMC controller is used to achieve a good trajectory tracking performance and guarantee the stability of the Quadrotor system. The main contributions of this paper are listed as follows:

1. A nonlinear control framework based on TSMC is developed for the Quadrotor system to achieve the finite-time convergence of both position and attitude tracking errors, performing a superior performance compared to conventional SMC.

2. To overcome the limitations of traditional SMC methods, RBFNNs are integrated as adaptive estimators to approximate the unknown dynamics, model uncertainties, and external disturbances without prior knowledge of system dynamics.

The rest of this paper is organized as follows. In Section 2, the details of the Quadrotor dynamic model and TSMC-based RBFNNs control design are presented. In Section 3, the simulation result is provided. Finally, Section 4 concludes this paper.

2. Problem Formulation and Control Design

2.1. Problem formulation

2.1.1. The dynamic model of a Quadrotor

The coordination and motion of the Quadrotor unmanned aerial vehicle are shown in Figure 1. Let $\boldsymbol{E} = \{X_E, Y_E, Z_E\}$ denotes the inertial Earth-fixed frame and $\boldsymbol{B} = \{X_B, Y_B, Z_B\}$ denotes the body-fixed frame attached to the Quadrotor. Let $\eta = [x, y, z]^T \in \boldsymbol{R}^3$ denotes the position of the Quadrotor mass center in the inertial Earth-fixed frame and the Euler angles (i.e., roll, pitch, and roll angles) are represented by $\xi = [\phi, \theta, \psi]^T$, satisfying $0 < \phi < \pi/2$, $0 < \theta < \pi/2$, and $-\pi < \psi < \pi$.

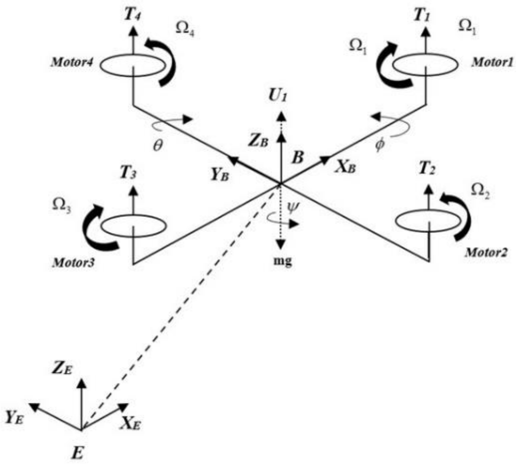

Figure 1. *The Quadrotor coordinate*

According to [19], the dynamic model of the Quadrotor is described by

$$\begin{aligned} \ddot{x} &= -\frac{k_x}{m}\dot{x} + \frac{1}{m}\left(c_\phi c_\psi s_\theta + s_\phi s_\psi\right)u_T + d_x \\ \ddot{y} &= -\frac{k_y}{m}\dot{y} + \frac{1}{m}\left(c_\phi s_\psi s_\theta - s_\phi c_\psi\right)u_T + d_y \\ \ddot{z} &= -\frac{k_z}{m}\dot{z} + \frac{1}{m}\left(c_\phi c_\theta\right)u_T - g + d_z \end{aligned} \tag{1a}$$

$$\begin{aligned} \ddot{\phi} &= \frac{1}{J_\phi}\left(\dot{\theta}\dot{\psi}\left(J_\theta - J_\psi\right) - J_r\dot{\theta}\bar{\Omega} - k_\phi\dot{\phi}^2 + u_\phi\right) + d_\phi \\ \ddot{\theta} &= \frac{1}{J_\theta}\left(\dot{\phi}\dot{\psi}\left(J_\psi - J_\phi\right) - J_r\phi\bar{\Omega} - k_\phi\dot{\theta}^2 + u_\theta\right) + d_\theta \\ \ddot{\psi} &= \frac{1}{J_\psi}\left(\dot{\phi}\dot{\theta}\left(J_\phi - J_\theta\right) - k_\psi\dot{\psi}^2 + U_\psi\right) + u_\psi \end{aligned} \tag{1b}$$

Where $\bar{\Omega} = \Omega_4 + \Omega_3 - \Omega_2 - \Omega_1$; J_r is the rotor inertia; d_i $(i = x, y, z, \phi, \theta, \psi)$ are time-varying external disturbance; $J = diag\left(\left[J_\phi, J_\theta, J_\psi\right]\right)$ is the inertial matrix. The total thrust u_T and the control torque $u_\xi = \left[u_\phi, u_\theta, u_\psi\right]^T$ are generated by the rotors, which are formulated by

$$\begin{aligned} u_T = k_T \textstyle\sum_{i=1}^{4}\Omega_i, \quad u_\phi = k_T l(\Omega_2 - \Omega_4), \quad u_\theta = k_T l(\Omega_1 - \Omega_3) \\ u_\psi = k_\tau \sum_{i=1}^{4}(-1)^{i+1}\Omega_i \end{aligned} \tag{2}$$

Where, $k_T > 0$ is the thrust coefficient ;$k_\tau > 0$ is the drag coefficient; l is arm length; and Ω_i is the speed of rotors. It is widely recognized that the parameters of Quadrotor system are uncertain. Therefore, adaptive control framwork is cruial to achieve good tracking performance corresponding to a predefined trajectory.

2.1.2. Model transformation

It is easy to observe from the Quadrotor dynamic model (1a) and (1b) that the Quadrotor is an underactuated system, having six degrees of freedom while having only four independent control inputs. To overcome this limitation, a hierarchical control structure, including a position controller and an attitude controller, is employed. Firstly, the virtual position control input $u_\eta = \left[u_x, u_y, u_z\right]^T$ is defined as the following

$$u_x = (c_\phi c_\psi s_\theta + s_\phi s_\psi) u_T$$
$$u_y = (c_\phi s_\psi s_\theta - s_\phi c_\psi) u_T \quad (3)$$
$$u_z = (c_\phi c_\theta) u_T$$

The dynamic model of Quadrotor (5) can be rewritten as follows

$$\ddot{\eta} = -k_p \dot{\eta} + m^{-1} u_\eta - g z_I + d_\eta \quad (4)$$
$$\ddot{\xi} = f_\xi(\xi, \dot{\xi}) + J^{-1} u_\xi + d_\xi$$

Where $z_I = [0,0,1]^T, d_\eta = [d_x, d_y, d_z]^T$, $d_\xi = [d_\phi, d_\theta, d_\psi]^T$, and the term $f_\xi(\xi, \dot{\xi})$ is described as

$$f_\xi(\xi, \dot{\xi}) = \begin{bmatrix} \frac{1}{J_\phi}\left(\dot{\theta}\dot{\psi}(J_\theta - J_\psi) - J_r \dot{\theta}\overline{\Omega} - k_\phi \dot{\phi}^2\right) \\ \frac{1}{J_\theta}\left(\dot{\phi}\dot{\psi}(J_\psi - J_\phi) - J_r \dot{\phi}\,\overline{\Omega} - k_\phi \dot{\theta}^2\right) \\ \frac{1}{J_\psi}\left(\dot{\phi}\dot{\theta}(J_\phi - J_\theta) - k_\psi \dot{\psi}^2 + U_\psi\right) \end{bmatrix} \quad (5)$$

Assumption 1. *The external disturbances acting on the Quadrotor UAV, arising from factors such as wind, aerodynamic drag, and friction, are assumed to be bounded along with their time derivatives in this study [20].*

Once the virtual control input u_η is determined, the desired reference for the attitude controller $\xi_d = [\phi_d, \theta_d, \psi_d]^T$ is derived as

$$u_T = \sqrt{u_x^2 + u_y^2 + u_\eta^2}$$
$$\phi_d = \arcsin\left(\frac{u_x \sin(\psi_d) - u_y cos(\psi_d)}{u_T}\right) \quad (6)$$
$$\theta_d = \arctan\left(\frac{u_x \cos(\psi_d) + u_y sin(\psi_d)}{u_z}\right)$$

It is noted that the desired yaw angle ψ_d is usually set as a fixed constant.

2.2. Control Design

This section presents an adaptive neural network-based terminal sliding mode control (ANN-based TSMC) for both position and attitude controllers of the Quadrotor UAVs. The position controller is designed to generate a virtual position control signal u_η tracks the position of the Quadrotor UAV with the desired reference trajectory. Then, the attitude controller is designed to generate the desired control signal u_ξ to achieve the stability of the Quadrotor. Based on the Lyapunov stability theory, the stability of the closed-loop system is proved.

2.2.1. Position control design

Let $\boldsymbol{p_d} = [\boldsymbol{x_d}, \boldsymbol{y_d}, \boldsymbol{z_d}]^{\boldsymbol{T}} \in \boldsymbol{R^3}$ as the desired position reference. Define the position-tracking error as

$$e_\eta = \eta - \eta_d \quad (7)$$

The terminal sliding mode surface for the position controller is defined as

$$s_\eta = \dot{e}_\eta + \alpha_\eta |e_\eta|^{\beta_\eta} sign(e_\eta) \quad (8)$$

Where $\alpha_\eta > 0$ and $0 < \beta_\eta < 1$. The virtual model-based position control input is designed as

$$u_p = -m\left(c_{\eta 1} s_\eta + c_{\eta 2} \mathrm{sign}(s_\eta) - k_p \dot{\eta} - g z_I + \hat{d}_\eta - \ddot{\eta}_d + \alpha_\eta \beta_\eta |e_\eta|^{\beta_\eta - 1}\right) \quad (9)$$

Where $\hat{d}_\eta$ is the estimation disturbance of d_η. Similar to [17], due to the unknown disturbances and uncertainties in the position subsystem, the RBF NN is introduced to approximate these terms as

$$W_\eta^T \Phi(z_\eta) + \epsilon_\eta(z_\eta) = -k_p \dot{\eta} + d_\eta - \ddot{\eta}_d + \alpha_\eta \beta_\eta |e_\eta|^{\beta_\eta - 1} \tag{10}$$

where $z_\eta = [\eta, \dot{\eta}, \eta_d, \dot{\eta}_d, \ddot{\eta}_d]^T \in \boldsymbol{R}^{15}$ is the input vector, W_η is the ideal position weight matrix, $\Phi(z_\eta) = \left[\Phi_1(z_\eta), \dots, \Phi_{l_\eta}(z_\eta)\right]^T \in \boldsymbol{R}^{l_\eta}$ is chosen as the Gaussian function with are the width of the Gaussian function μ_η and the center of the receptive file c_η, and $\epsilon_\eta(z_\eta)$ is the approximate error which satisfies $|\epsilon_\eta(z_\eta)| \le \bar{\epsilon}_\eta$ with $\bar{\epsilon}_\eta$ is a possitive constant. The position control input is rewritten as

$$u_\eta = -m\left(\widehat{W}_\eta^{\mathrm{T}} \Phi(z_\eta) + c_{\eta 1} s_\eta + c_{\eta 2} \operatorname{sign}(s_\eta) - g z_I\right) \tag{11}$$

Where $\widehat{W}_\eta \in \boldsymbol{R}^{l_\eta}$ is the estimation value of the ideal position weight matrix. Then, the adaptive law is formulated as

$$\dot{\widehat{W}}_\eta = -\Phi(z_\eta) s_\eta^T \tag{12}$$

Theorem 1. Let $\widetilde{W}_\eta = W_\eta - \widehat{W}_\eta$ as the approximate weight error. For the position subsystem in (1a), the position control law (11) with the adaptive law (12), the position tracking error e_η is asymptotic stability.

Proof. Choosing a Lyapunov candidate function as

$$L_\eta = \frac{1}{2} s_\eta^T s_\eta + \frac{1}{2} tr\{\widetilde{W}_\eta^T \widetilde{W}_\eta\} \tag{13}$$

Taking the time derivative of L_η, we have

$$\begin{aligned}
\dot{L}_\eta &= s_\eta^T \dot{s}_\eta + tr\left\{\widetilde{W}_\eta^T \dot{\widetilde{W}}_\eta\right\} \\
&= s_\eta^T \left(-k_p \dot{\eta} + m^{-1} u_\eta - g z_I + d_\eta - \ddot{\eta} + \alpha_\eta \beta_\eta |e_\eta|^{\beta_\eta - 1}\right) + tr\{\widetilde{W}_\eta^T \dot{W}_\eta\} \\
&= s_\eta^T \big(-\widehat{W}_\eta^{\mathrm{T}} \Phi(z_\eta) - c_{\eta 1} s_\eta + c_{\eta 2} \operatorname{sign}(s_\eta) + g z_I + W_\eta^T \Phi(z_\eta) + \epsilon_\eta(z_\eta) \\
&\qquad - g z_I\big) - s_\eta^T \widetilde{W}_\eta^T \Phi(z_\eta) \\
&= -c_{\eta 1} s_\eta^T s_\eta - c_{\eta 2} s_\eta^T sign(s_\eta) - s_\eta^T \epsilon_\eta(z_\eta) \\
&\le -c_{\eta 1} s_\eta^T s_\eta - (c_{\eta 2} - \bar{\epsilon}_\eta)|s_\eta|
\end{aligned} \tag{14}$$

By choosing $c_{\eta 2} > \bar{\epsilon}_\eta$, the stability of the position subsystem is proven.

2.2.2. Attitude control design

After the virtual position control input is determined, the desired reference ξ_d for the attitude controller can be obtained by using (6). Define the attitude-tracking error e_ξ as

$$e_\xi = \xi - \xi_d \tag{15}$$

The terminal sliding mode surface for the attitude controller is defined as

$$s_\xi = \dot{e}_\xi + \alpha_\xi |e_\xi|^{\beta_\xi} sign(e_\xi) \tag{16}$$

Where $\alpha_\eta > 0$ and $0 < \beta_\eta < 1$. The model-based attitude control input is designed as

$$u_\xi = -J\left(c_{\xi 1} s_\xi + c_{\xi 2} \operatorname{sign}(s_\xi) + f_\xi(\xi, \dot{\xi}) + \hat{d}_\xi - \ddot{\xi}_d + \alpha_\xi \beta_\xi |e_\xi|^{\beta_\xi - 1}\right) \tag{17}$$

where $\hat{d}_\xi$ is the estimation disturbance of d_ξ. Similar to the position control design, the RBF NN is employed to approximate these terms as

$$W_\xi^T \Phi(z_\xi) + \epsilon_\xi(z_\xi) = f_\xi(\xi, \dot{\xi}) + d_\xi - \ddot{\xi}_d + \alpha_\xi \beta_\xi |e_\xi|^{\beta_\xi - 1} \tag{18}$$

Where $z_\xi = [\xi, \dot{\xi}, \xi_d, \dot{\xi}_d, \ddot{\xi}_d]^T$ is the input vector; W_ξ is the ideal attitude weight matrix; $\Phi(z_\xi) = \left[\Phi_1(z_\xi), \dots, \Phi_{l_\xi}(z_\xi)\right]^T \in \boldsymbol{R}^{l_\xi}$ is chosen as the Gaussian function with are the width of the Gaussian function μ_ξ; and the center of the receptive file c_ξ, and $\epsilon_\xi(z_\xi)$ is the approximate error which satisfies $\left|\epsilon_\xi(z_\xi)\right| \le \bar{\epsilon}_\xi$ with $\bar{\epsilon}_\xi$ is a possitive constant. The attitude control input is rewritten as

$$u_\xi = -J\left(\widehat{W}_\xi^{\mathrm{T}}\Phi(z_\xi) + c_{\xi 1}s_\xi + c_{\xi 2}\mathrm{sign}(s_\xi)\right) \tag{19}$$

Where $\widehat{W}_\xi$ is the estimation value of the ideal position weight matrix. Then, the adaptive law is formulated as

$$\dot{\widehat{W}}_\xi = -\Phi(z_\xi)s_\xi^T \tag{20}$$

Theorem 2. Let $\widetilde{W}_\xi = W_\xi - \widehat{W}_\xi$ as the approximate weight error. For the attitude subsystem in (1b), the attitude control law (19) with the adaptive law (20), the attitude tracking error e_ξ is asymptotic stability.

Proof. Choosing a Lyapunov candidate function as

$$L_\xi = \frac{1}{2}s_\xi^T s_\xi + \frac{1}{2}tr\{\widetilde{W}_\xi^T \widetilde{W}_\xi\} \tag{21}$$

Taking the time derivative of L_η, we have

$$\begin{aligned}
\dot{L}_\xi &= s_\xi^T \dot{s}_\xi + tr\left\{\widetilde{W}_\eta^T \dot{\widetilde{W}}_\eta\right\} \\
&= s_\xi^T\left(f_\xi(\xi,\dot{\xi}) + J^{-1}u_\xi + d_\xi - \ddot{\eta}_d + \alpha_\xi\beta_\xi|e_\xi|^{\beta_\xi - 1}\right) + tr\{\widetilde{W}_\xi^T \dot{W}_\xi\} \\
&= s_\xi^T\left(-\widehat{W}_\xi^{\mathrm{T}}\Phi(z_\xi) - c_{\xi 1}s_\xi + c_{\xi 2}\mathrm{sign}(s_\xi) + W_\xi^T\Phi(z_\xi) + \epsilon_\xi(z_\xi)\right) \\
&\qquad - s_\xi^T \widetilde{W}_\xi^T \Phi(z_\eta) \\
&= -c_{\xi 1}s_\xi^T s_\xi - c_{\xi 2}s_\xi^T sign(s_\xi) - s_\xi^T \epsilon_\xi(z_\xi) \\
&\le -c_{\xi 1}s_\xi^T s_\xi - (c_{\xi 2} - \bar{\epsilon}_\xi)|s_\xi|
\end{aligned} \tag{22}$$

By choosing $c_{\xi 2} > \bar{\epsilon}_\xi$, the stability of the position subsystem is proven.

3. Simulation result

In this section, a simulation result is provided to verify the effectiveness of the proposed controller. The parameters of the Quadrotor system are chosen in [19]. The parameters of the Quadrotor system are shown in **Table 1.** The position and attitude control coefficients are chosen in **Table 2**. The initial condition of the Quadrotor is configurated as follows $\eta(0) = [0.5, 0.5, 0.5]^T$, $\dot{\eta}(0) = [0.0, 0.0, 0.0]^T$, $\xi(0) = [0.0, 0.0, 0.0]^T$, and $\dot{\xi}(0) = [0.0, 0.0, 0.0]^T$.

Table 1. *The parameter of the Quadrotor system*

Parameter	Value	Parameter	Value
g	$9.81\ m/s^2$	k_x	$5.567 \times 10^{-4}\ Ns/m$
m	$0.74\ kg$	k_y	$5.567 \times 10^{-4}\ Ns/m$
J_ϕ	$4 \times 10^{-3}\ kgm^2$	k_z	$5.567 \times 10^{-4}\ Ns/m$
J_θ	$4 \times 10^{-3}\ kgm^2$	k_ϕ	$5.567 \times 10^{-4}\ Ns/rad$
J_ψ	$8.4 \times 10^{-3}\ kgm^2$	k_θ	$5.567 \times 10^{-4}\ Ns/rad$
J_r	$2.8385 \times 10^{-5}\ kgm^2$	k_ψ	$5.567 \times 10^{-4}\ Ns/rad$
k_T	1	l	$0.2\ m$
k_τ	1		

Table 2. *The parameters of the position and attitude controllers*

Parameter	Value	Parameter	Value
α_η	1.1	α_η	1.1
β_η	0.5	β_η	0.5
$c_{\eta 1}$	15.0	$c_{\eta 1}$	15.0
$c_{\eta 2}$	0.5	$c_{\eta 2}$	0.5

The desired trajectory reference is set as follows $p_d = [0.5\sin(0.5t), 2\cos(0.5t), 2.0 + t]$. The external disturbances are assumed as $d_x = d_y = d_z = 0.1\sin(t)$ and $d_\phi = d_\theta = d_\psi = 0.1\sin(0.5t)$ .The desired yaw angle is predefined as $\psi_d = 0.0$. The RBF NN in both position and attitude controllers are designed with 5 and 7 nodes, respectively. The width of the Gaussian function $\mu_\eta = \mu_\xi = 0.4$, and the center c_η and c_ξ are evenly distributed in $[-2, 2]$ and $[-3, 3]$, respectively. The initial weight matrices for position and attitude controllers are chosen as $W_\eta(0) = 0_{5\times3}$ and $W_\xi(0) = 0_{7\times3}$, respectively.

The convergence of RBF NN for position and attitude controllers are shown in Figure 2 And Figure 3, respectively. The position and attitude responses of the Quadrotor are shown in Figure 4 and Figure 5, respectively. After 2 seconds, both the position and attitude states of the Quadrotor track follow the position and attitude references. The "chattering" seen in the attitude tracking results of Figure 5 originates from an inherent limitation of the TSMC scheme. This issue can be alleviated either by smoothing the switching term - replacing the discontinuous sign(·) function with continuous functions such as tanh(·) or sat(·) functions - or by optimising the control gains with meta heuristic techniques like Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) to obtain coefficient sets that suppress chattering without sacrificing robustness. This aspect will be further discussed in future work, as it is not the focus of the current study. To clarify the effectiveness of the proposed controller, a 3D trajectory is provided in Figure 6.

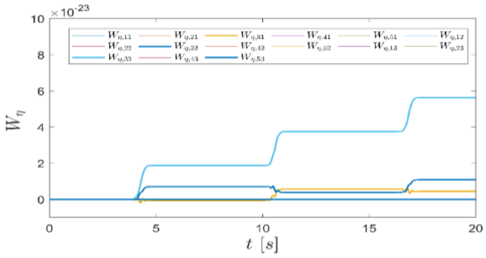

Figure 2. *The convergence of RBFNN in the position controller*

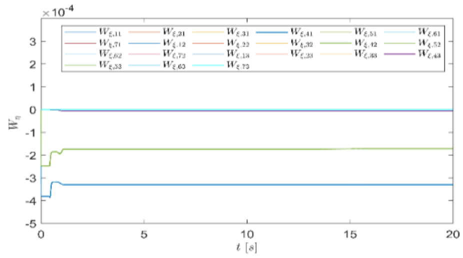

Figure 3. *The convergence of RBF NN in the attitude controller*

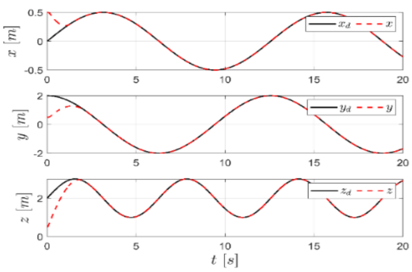

Figure 4. *The position responses of the Quadrotor*

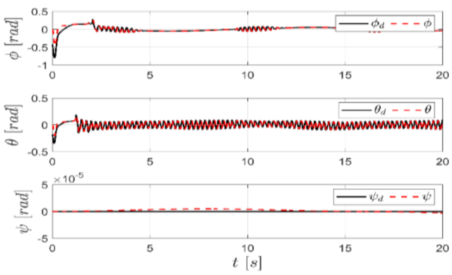

Figure 5. *The attitude responses of the Quadrotor*

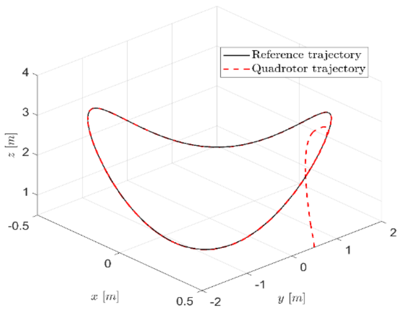

Figure 6. *The 3D trajectory of the Quadrotor*

4. Conclusion

The study proposes an adaptive neural network-based terminal sliding mode strategy for the trajectory tracking control for the Quadrotor subjected to unknown external disturbances. Based on the terminal sliding mode control, the position and attitude tracking error converge to zero in the finite time. However, both controllers cannot be obtained directly due to model uncertainties and external disturbances. Therefore, the RBF NN is employed to approximate the uncertain dynamics of the Quadrotor and the external disturbances. The simulation result is carried out to clarify the superior performance of the proposed controller. It is noted that this paper does not consider the optimization of the quadrotor trajectory-tracking problem. In practical applications, due to the limited battery capacity, the problem of optimal control for a single or multiple quadrotor systems should be taken into consideration and remains a subject for future work.

REFERENCES

[1] D. K. D. Villa, A. S. Brandão, and M. Sarcinelli-Filho, "A Survey on Load Transportation Using Multirotor UAVs," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 98, no. 2, pp. 267–296, May 2020, doi: 10.1007/S10846-019-01088-W/METRICS.

[2] M. Lyu, Y. Zhao, C. Huang, and H. Huang, "Unmanned Aerial Vehicles for Search and Rescue: A Survey," *Remote Sensing 2023, Vol. 15, Page 3266*, vol. 15, no. 13, Jun. 2023, Art. no. 3266, doi: 10.3390/RS15133266.

[3] W. Budiharto, E. Irwansyah, J. S. Suroso, A. Chowanda, H. Ngarianto, and A. A. S. Gunawan, "Mapping and 3D modelling using Quadrotor drone and GIS software," *J. Big Data*, vol. 8, no. 1, pp. 1–12, Dec. 2021, doi: 10.1186/S40537-021-00436-8/FIGURES/7.

[4] M. Sivakumar and T. Y. J. N. Malleswari, "A Literature Survey of Unmanned Aerial Vehicle Usage for Civil Applications," *Journal of Aerospace Technology and Management*, vol. 13, Nov. 2021, Art. no. e4021, doi: 10.1590/JATM.V13.1233.

[5] I. Lopez-Sanchez and J. Moreno-Valenzuela, "PID control of quadrotor UAVs: A survey," *Annu Rev Contro*l, vol. 56, Jan. 2023, Art. no. 100900, doi: 10.1016/J.ARCONTROL.2023.100900.

[6] J. Lin, Z. Miao, Y. Wang, G. Hu, X. Wang, and H. Wang, "Error-State LQR Geofencing Tracking Control for Underactuated Quadrotor Systems," *IEEE/ASME Transactions on Mechatronics*, vol. 29, no. 2, pp. 1146–1157, Apr. 2024, doi: 10.1109/TMECH.2023.3292893.

[7] W. Xie, D. Cabecinhas, R. Cunha, and C. Silvestre, "Adaptive Backstepping Control of a Quadcopter with Uncertain Vehicle Mass, Moment of Inertia, and Disturbances," *IEEE Transactions on Industrial Electronics*, vol. 69, no. 1, pp. 549–559, Jan. 2022, doi: 10.1109/TIE.2021.3055181.

[8] C. A. I. Zhenhuan, S. Zhang, and X. Jing, "Model predictive controller for quadcopter trajectory tracking based on feedback linearization," *IEEE Access*, vol. 9, pp. 162909–162918, 2021, doi: 10.1109/ACCESS.2021.3134009.

[9] X. Liu, Z. Yuan, Z. Gao, and W. Zhang, "Reinforcement Learning-Based Fault-Tolerant Control for Quadrotor UAVs Under Actuator Fault," *IEEE Trans Industr. Inform*, vol. 20, no. 12, pp. 13926–13935, Sep. 2024, doi: 10.1109/TII.2024.3438241.

[10] G. Han, O. Mofid, S. Mobayen, and M. H. Khooban, "Adaptive prescribed performance based on recursive nonsingular terminal sliding mode control for quad-rotor systems under uncertainty and disturbance: Real-time validation," *Aerosp Sci. Technol*, vol. 147, Apr. 2024, Art. no. 109028, doi: 10.1016/J.AST.2024.109028.

[11] O. Mechali, L. Xu, Y. Huang, M. Shi, and X. Xie, "Observer-based fixed-time continuous nonsingular terminal sliding mode control of Quadrotor aircraft under uncertainties and disturbances for robust trajectory tracking: Theory and experiment," *Control Eng Pract*, vol. 111, Jun. 2021, Art. no. 104806, doi: 10.1016/J.CONENGPRAC.2021.104806.

[12] S. J. Gambhire, D. R. Kishore, P. S. Londhe, and S. N. Pawar, "Review of sliding mode based control techniques for control system applications," *Int. J. Dyn. Control*, vol. 9, no. 1, pp. 363–378, Mar. 2021, doi: 10.1007/S40435-020-00638-7/METRICS.

[13] Y. Mousavi, G. Bevan, I. B. Kucukdemiral, and A. Fekih, "Sliding mode control of wind energy conversion systems: Trends and applications," *Renewable and Sustainable Energy Reviews*, vol. 167, Oct. 2022, Art. no. 112734, doi: 10.1016/J.RSER.2022.112734.

[14] J. Hu, H. Zhang, H. Liu, and X. Yu, "A survey on sliding mode control for networked control systems," *Int. J. Syst. Sci.*, vol. 52, no. 6, pp. 1129–1147, Apr. 2021, doi: 10.1080/00207721.2021.1885082.

[15] X. Lin, Y. Wang, and Y. Liu, "Neural-network-based robust terminal sliding-mode control of Quadrotor," *Asian J. Control*, vol. 24, no. 1, pp. 427–438, Jan. 2022, doi: 10.1002/ASJC.2478.

[16] Q. Liu, D. Li, S. S. Ge, R. Ji, Z. Ouyang, and K. P. Tee, "Adaptive bias RBF neural network control for a robotic manipulator," *Neurocomputing*, vol. 447, pp. 213–223, Aug. 2021, doi: 10.1016/J.NEUCOM.2021.03.033.

[17] H. Pang, M. Liu, C. Hu, and F. Zhang, "Adaptive sliding mode attitude control of two-wheel mobile robot with an integrated learning-based RBFNN approach," *Neural Comput. Appl.*, vol. 34, no. 17, pp. 14959–14969, Sep. 2022, doi: 10.1007/S00521-022-07304-3/METRICS.

[18] R. Ma, J. Han, and L. Ding, "Finite-time trajectory tracking control of Quadrotor UAV via adaptive RBF neural network with lumped uncertainties," *Mathematical Biosciences and Engineering*, vol. 20, no. 2, pp. 1841–1855, 2022, doi: 10.3934/mbe.2023084.

[19] M. Labbadi and M. Cherkaoui, "Robust adaptive nonsingular fast terminal sliding-mode tracking control for an uncertain Quadrotor UAV subjected to disturbances," *ISA Trans*, vol. 99, pp. 290–304, Apr. 2020, doi: 10.1016/J.ISATRA.2019.10.012.

[20] J.Xu, P. Shi, C. C. Lim, C. Cai, and Y. Zou, "Reliable Tracking Control for Under-Actuated Quadrotors with Wind Disturbances," *IEEE Trans Syst Man Cybern Syst*, vol. 49, no. 10, pp. 2059–2070, Oct. 2019, doi: 10.1109/TSMC.2017.2782662.

[21] Y. Hu, H. Yan, H. Zhang, M. Wang, and L. Zeng, "Robust Adaptive Fixed-Time Sliding-Mode Control for Uncertain Robotic Systems with Input Saturation," *IEEE Trans Cybern*, vol. 53, no. 4, pp. 2636–2646, Apr. 2023, doi: 10.1109/TCYB.2022.3164739.