DETERMINE THE DAMAGE BOUNDARY CURVE FOR THE PACKAGING USING THE FINITE ELEMENT METHOD

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ABSTRACT

The damage boundary curve is an important criterion in the packaging design process, especially in assessing the deformation resistance of paper packaging. This study presents a method for quickly determining the damage boundary curve using the finite element method, directly serving the needs of design and optimization of the forming process in the packaging industry. Based on the elastic-plastic homogenization method developed for corrugated core cardboard, an equivalent model for carton boxes subjected to vibration is built. This model significantly reduces the computation time compared to the homogeneous model. Thereby helping to determine the failure limit curve of carton packaging quickly. The method is verified by comparing it with experimental data and shows high accuracy, which is suitable for the strict requirements of high-speed packaging production. The research results open up an effective and time-saving approach in applying the finite element method to the analysis, evaluation, and control of damage in the packaging industry in general.

XÁC ĐỊNH ĐƯỜNG CONG RANH GIỚI PHÁ HỦY CHO BAO BÌ BẰNG PHƯƠNG PHÁP PHẦN TỬ HỮU HẠN

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TỪ KHÓA

Đường ranh giới phá hủy
Phần tử hữu hạn
Va chạm
Rung động
Mô phỏng

TÓM TẮT

Đường ranh giới phá hủy là một tiêu chí quan trọng trong quy trình thiết kế bao bì, đặc biệt trong việc đánh giá khả năng chịu biến dạng của bao bì bằng vật liệu giấy. Nghiên cứu này trình bày một phương pháp xác định nhanh đường ranh giới phá hủy bằng phương pháp phần tử hữu hạn, phục vụ trực tiếp cho nhu cầu thiết kế và tối ưu hóa quá trình tạo hình trong công nghiệp bao bì. Dựa trên phương pháp đồng nhất hóa đàn hồi-dẻo đã phát triển cho tấm carton lõi sóng, một mô hình tương đương cho hộp carton chịu rung động được xây dựng. Mô hình sẽ giúp giảm thời gian tính toán từ đó giúp xác định nhanh đường cong giới hạn phá hủy của hộp carton. Phương pháp được kiểm chứng bằng cách so sánh với dữ liệu thực nghiệm và cho thấy độ chính xác cao, phù hợp với yêu cầu khắt khe của sản xuất bao bì tốc độ cao. Kết quả nghiên cứu mở ra hướng tiếp cận hiệu quả và tiết kiệm thời gian trong việc ứng dụng phương pháp phần tử hữu hạn vào kiểm soát phá hủy trong ngành bao bì nói chung.

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1. Introduction

In the packaging industry, cardboard boxes are among the most popular packaging types today due to their flexibility, low cost, and high recyclability. During storage and transportation, cardboard boxes often undergo mechanical impacts such as compression, bending, twisting, or impact, which can easily lead to deformation and damage to both the packaging and the products within. To ensure the quality and durability of packaging, accurately assessing the load-bearing capacity and determining the damage limits are critical issues in the design and development of packaging systems. To date, numerous publications have addressed the determination of packaging durability [1] – [5]. To quantify and analyze the vibrations occurring during transportation, sensors have been installed on transport vehicles, packages, or products to record vibration data throughout the entire logistics cycle. Some typical random signals corresponding to white noise and the package’s response have been recorded [6]. The recorded vibrations are variable and unpredictable, making it challenging to predict, analyze, or synthesize them accurately. The most common approach involves calculating the average vibration power spectral density (PSD), defined as the square of the Fourier transform modulus divided by the spectral bandwidth, which itself is equal to the inverse of the integration time T. This technique is beneficial in various ways, such as determining the overall frequency and overall vibration level using the root mean square (RMS) value.

Previous studies have analyzed the vibration levels during freight transport on road and rail networks to propose PSDs for each type of transport [7] – [11]. Subsequently, testing methods for packaging systems based on this data were proposed. Low-frequency vibrations (0.1–10 Hz) were found to be much more harmful to the product than high-frequency vibrations [12]. Using cumulative distribution functions (CDFs) of the RMS values of time-domain vibrations measured in the horizontal, vertical, and longitudinal directions, Pateroster et al. [13] analyzed the vibration levels of cardboard packaging during beer truck transportation in Belgium. Vibration tests at different frequencies of cardboard boxes containing agricultural products such as apples [14] and bananas [15] were carried out to evaluate the performance of the boxes. Vibration experiments on cardboard packaging were also carried out in the laboratory using a vibrating table instead of conducting direct tests during actual transportation. Jamialahmadi [16] performed dynamic tests by placing packaging systems on a vibrating table. The above studies generally used direct experiments on the packaging.

To optimize packaging design, criteria for evaluating load-bearing and failure limits are increasingly of interest, with the damage boundary curve (DBC) is an effective tool to describe the load-bearing capacity of packaging structures under various stress combinations. Additionally, the Finite Element Method (FEM) has been applied to assess the durability of packaging by determining the DBC curve [17]. This is a powerful analysis tool that allows for detailed simulations of the mechanical behavior of materials and complex packaging geometries. However, determining DBC through FEM is often time-consuming, particularly during the initial design or product optimization stages. In this study, we propose a FEM model to determine the DBC failure boundary for carton boxes. This method reduces computation time, allowing for rapid determination of the DBC.

2. Research methods

2.1. Basis for constructing the damage boundary curve

The damage boundary curve is a diagram that illustrates the boundary separating the two regions of failure and non-failure states of a material or structure. In the analysis of materials mechanics, particularly in problems subjected to impact or dynamic loading, the DBC enables the determination of the minimum boundary condition that causes damage to the member. An accurate determination of the DBC is crucial for predicting the behavior of the member under load and establishing safety limits in the design. Constructing of the DBC curve typically relies on data gathered from a series of experiments or numerical simulations.

The basis for constructing the DBC curve is derived from the dynamic model of the product [2] as a system, as shown in Figure 1. When the product is subjected to an impact from a vertical shock tester, the impulse load is transmitted through the forced movement of the shock table, denoted z(t), where the test specimen is fixed. The dynamic response y(t) of the mass is excited by the force transmitted through the elastic element with linear stiffness K.

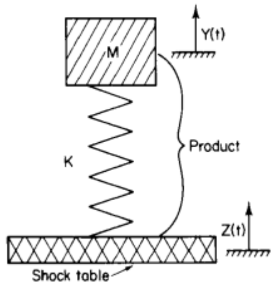

Figure 1. *Spring/mass model of product on shock table [2]*

Considered in a static reference system, with y(t) and z(t) measured relative to the initial equilibrium position, the differential equation controlling the system's motion is established based on Newton's second law, leading to the standard form of the excited harmonic oscillation equation.

$$\frac{d^2y}{dt^2} + w^2 y = w^2 z \qquad (1)$$

$$\frac{d^2x}{dt^2} + w^2 x = \frac{d^2z}{dt^2} \qquad (2)$$

$$x(t) = x_0 \cos wt + \frac{\dot{x}_0}{w} \sin wt + \frac{1}{w}\int_0^t A(s) \sin w(t-s) ds \qquad (3)$$

The acceleration of the mass from equations (1) and (3) is

$$a(t) = \frac{d^2y}{dt^2} = w^2 x = w\int_0^t A(s) \sin w(t-s) ds \qquad (4)$$

The result obtained is the DBC limit curve for the spring/mass model as shown in Figure 2. Where V_{cr} is the change in critical velocity, and A_{cr} is the critical acceleration.

An alternative damage boundary curve to the rigid/fully plastic model was then developed and shown to be a better estimate of brittleness for a broader group of products based on the results of shock tests. DBC is applied to predict and limit product damage when designing packaging and planning impact tests.

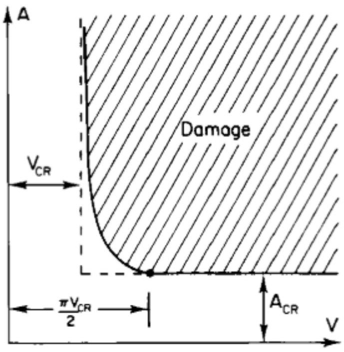

Figure 2. *DBC for spring/mass model [2]*

2.2. Basis for constructing the equivalent model

This study focuses on creating an equivalent model for corrugated core board. The model is then utilized to build corrugated core board boxes. This model helps reduce the time to determine the DBC failure boundary. Luong et al. [18] developed an homogenization model (H-2D) that fully describes the elastic-plastic behavior of corrugated core cardboard panels. This helps to significantly reduce the calculation time while still ensuring the accuracy of the results. Based on practical conditions during goods transportation, corrugated cardboard boxes are often stacked into pallets (Figure 3) and are subjected to vertical vibrations. A vibration resistance test for the cardboard boxes was conducted [17]. The box positioned at the bottom of the pallet was selected for testing, where the weight of the boxes above was replaced by masses of varying magnitudes, and the failure curve was determined as shown in Figure 4.

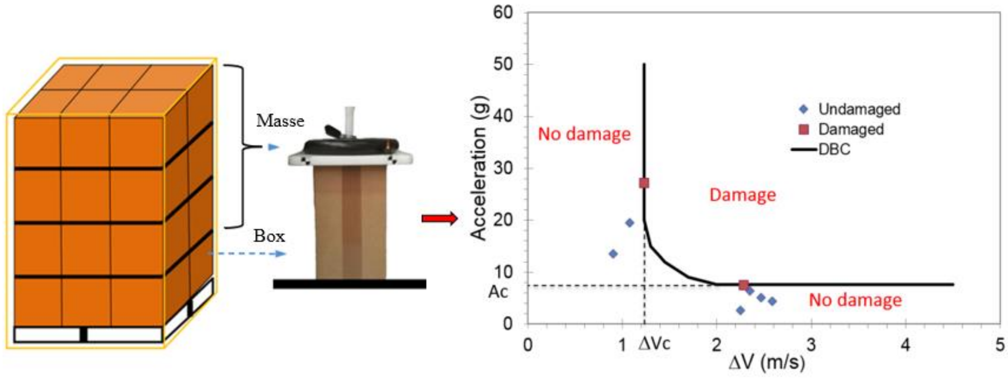

Figure 3. *Carton box modeling and experimental DBC curve [17]*

The H-2D model is employed to transform three-dimensional carton boards into two-dimensional flat configurations, which serve as the basis for constructing a finite element model (FEM) of the carton box using S4R shell elements (Figure 4). A simulation of a carton box vibration experiment was conducted to determine the DBC failure limit curve. This model significantly reduces calculation time compared to the 3D model and experiment (detailed in the following section). However, when applying the H-2D model, it is necessary to use the UGENS subroutine, a tool for calculating equivalent stiffness for cardboard panels, and the user must program the model and integrate other supporting subroutines. When simulating the vibration test of a cardboard box, each impact causes the box to undergo compression, subjecting the walls to vertical compression and bending (without the impact of horizontal shear forces) [12]. Based on that, the study proposes to build a new equivalent model, called the E-2D model, developed using the inverse identification method. When simulating a carton box subjected to vibration, the E-2D model only needs to use the VUMAT subroutine, which optimizes the calculation time of the failure curve.

The E-2D model construction process includes the following steps:

Step 1: From the 2D plate obtained by the homogenization method, tensile samples in three directions (MD, CD, and 45°) are created;

Step 2: Conduct tensile test simulations for these three samples. The results obtained are considered experimental data.

Step 3: Apply the inverse identification process [19] to determine the material parameters for the E-2D plate. Accordingly, the objective function is determined according to equation (5):

$$F_{obj} = \frac{1}{N}\sum_{i=1}^{N}\left(F_{num}\left(U_{num}\right) - F_{\exp}\left(U_{\exp}\right)\right)^2 \quad (5)$$

Where N is the number of data sets, U_{num} and U_{exp} are the numerical and experimental displacement, and F_{num} and F_{exp} are the tensile forces determined by numerical simulation and measured experimentally, respectively.

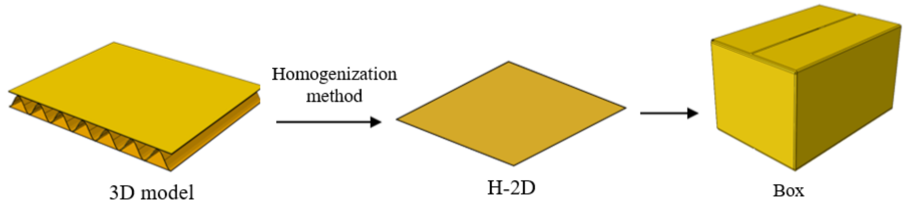

Figure 4. *Creating FEM model of carton box using homogenization method*

To improve the efficiency of the optimization process, especially the fast convergence speed, the MOGA-II algorithm is used to determine the values of parameters in the behavior model of cardboard. Finally, the E-2D model is applied to the carton box model to simulate the vibration test (Figure 5). The results and evaluation of the accuracy of the model are presented in the next section.

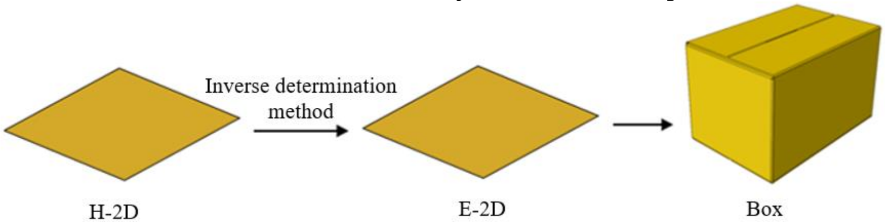

Figure 5. *Creating FEM model of carton box using homogenization method*

3. Results and discussion

In this section, the research material is C flute cardboard, with properties shown in Figure 6 and Table 1 [17]. Numerical simulations of tensile tests on 2D cardboard plates in MD, CD, and 45^0 directions were performed. The results are presented in Figure 7. The identification process using the objective function (5) yielded the material parameter values for the E-2D plate as shown in Table 2. Figure 8 illustrates that the material parameter values in the E-2D model lign well with the force-displacement curve of the H-2D model.

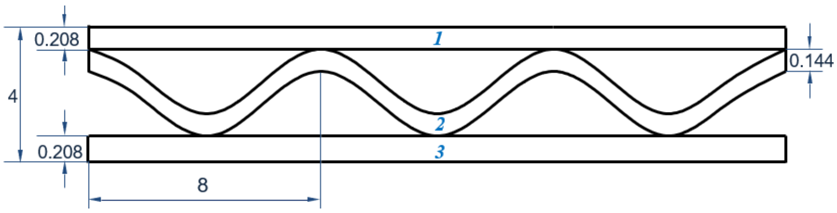

Figure 6. *Geometric dimensions of C flute cardboard [17]*

Table 1. *Parameters of the IPE model for corrugated cardboard constituent [17]*

Layer	E_x (MPa)	E_y (MPa)	ν_{xy}	G_{xy} (MPa)	E_0 (MPa)	n
1,3	2433.2	859.91	0.0829	1077.2	96.45	4.97
2	1130.4	625.85	0.0717	303.05	87.31	4.247
Layer	**A**	**B**	**C**	**D**	**ε_0**	
1,3	1.0	2.498	2.498	1.622	0.48e-3	
2	1.0	2.178	2.178	1.871	0.92e-3	

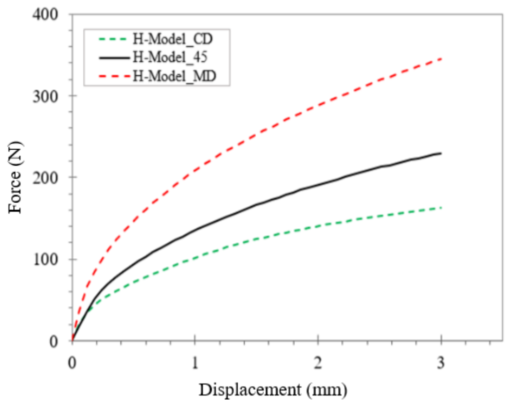

Figure 7. *Tensile force - elongation curve of H-2D plate specimen*

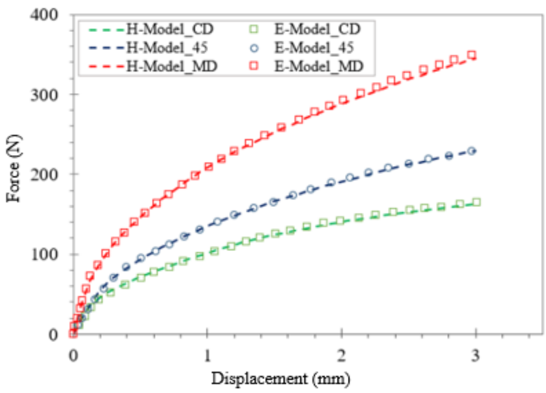

Figure 8. *Inverse force-time curve of E-2D plate*

Table 2. *Equivalent material parameters*

E_x (MPa)	E_y (MPa)	ν_{xy}	G_{xy} (MPa)	E_0	n	A	B	C	D	ε_0
2483.3	1322.7	0.348	488.3	157.6	3.56	1	2.04	2.049	2.21	0.0001

To verify the E-2D model, a numerical simulation experiment in the study [17] was conducted with all three carton models (3D, H-2D, and E-2D) subjected to a 6 kg load attached to a rigid upper plate. A trapezoidal excitation pulse (A = 10 g, t = 14 ms) was applied to the lower plate for a brief duration. Cardboard box dimensions: length = 222 mm, width = 185 mm, and height = 295 mm (Figure 9). During the simulation, the acceleration amplitude and velocity change of the top plate were recorded. The results obtained from the three models are compared and presented in Table 3. The findings indicate that the simulation with the 3D model takes a considerable time (14,163 seconds), while the H-2D model requires only 621 seconds, making it approximately 25.7 times faster. In particular, the E-2D model completes the task in just 195 seconds, which is 111 times faster than the 3D model and 3 times quicker than the H-2D model. In terms of value, all three models yield very similar results, demonstrating the high reliability of the E-2D model.

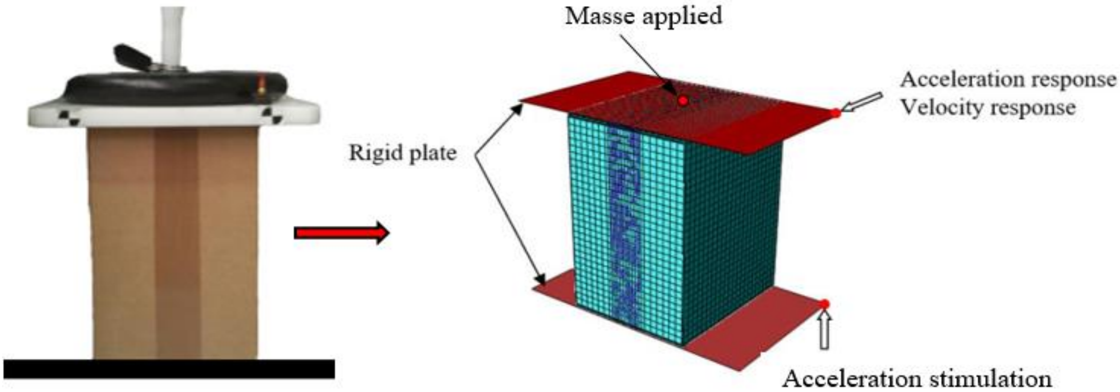

Figure 9. *Carton box model in Abaqus*

Table 3. *Comparison between 3D models, H-2D models and E-2D models*

Model	3D Model	H-2D Model	E-2D Model
A excitation (g)	10	10	10
A response (g)	20.8	19.1	19.5
Response velocity (m/s)	2.1	1.9	1.85
CPU time (s)	14163	621	195

Next, the E-2D model is used to determine the failure curve for the corrugated core carton box. Figure 10 compares the DBCs for the box subjected to the impact of an object with a mass of M=16.9 kg. It is evident that there is good agreement among these three curves, where the relative difference between the experimental results and the simulation results of the H model for the critical acceleration AC and the critical speed change ΔVC is less than 6% (Table 4). The deviation between the E-2D model and the experiment is under 7%. The critical acceleration and the change in critical speed are very important for making design decisions. Thus, based on the comparison of the obtained results, it is clear that the E-2D model proposed in this study can be utilized to quickly determine the DBC curve when designing carton boxes.

Table 4. *Comparison of acceleration and critical speed change between H-2D model, E-2D model and experiment*

	Experiment	H-2D Model	E-2D Model
A_C (g)	6.7	6.5	6.5
ΔV_C (m/s)	0.9	0.85	0.89

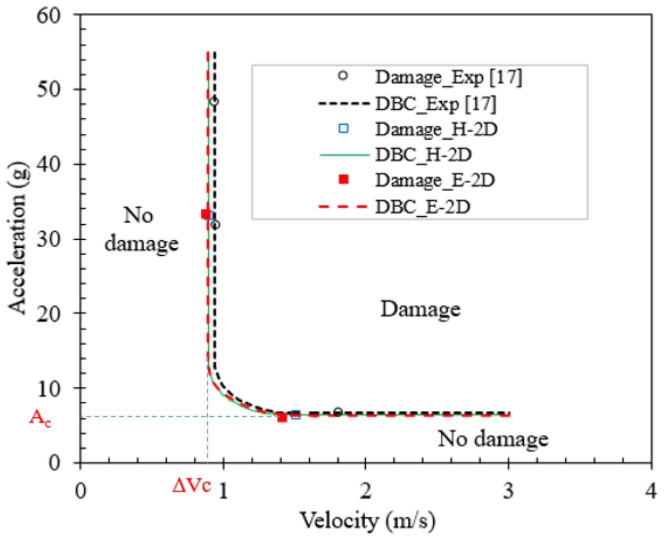

Figure 10. *Comparison of experimental DBC curve with numerical DBC curve of H-2D and E-2D models*

4. Conclusion

The Damage Boundary Curve is an effective tool for analyzing, evaluating, and predicting the damage state of structures under mechanical impact. Constructing the DBC based on experimental data or numerical simulation significantly improves design efficiency and ensures the safety of mechanical systems, especially in collision load problems. This study has developed a method for quickly determining the DBC based on numerical simulations using the finite element method, aiming to shorten calculation time and enhance efficiency in the design process. This study has contributed to clarifying the failure mechanism of packaging materials, especially multi-layer cartons, through the construction and analysis of the failure boundary. The application of the finite element method allows for accurate simulation of the nonlinear behavior of materials and the damage formation process, thereby providing a scientific basis for durability assessment and packaging structure design. Determining DBC by numerical simulation also supports enterprises in the packaging industry to improve products, develop technical standards and improve the quality of packaging goods. The research results show that this method can be extended to accommodate more complex packaging materials and geometries.

Acknowledgments

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