APPLICATION OF HOMOGENIZATION MODEL TO DETERMINE FATIGUE STRENGTH OF CARTON PACKAGING

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ABSTRACT

Corrugated cardboard packaging is widely used in the transportation, distribution, and storage of goods. It is frequently subjected to repeated mechanical impacts that can result in packaging failure. This paper presents a numerical approach utilizing homogenization modeling to construct a finite element model for determining the fatigue strength of corrugated cardboard. The computational cost was significantly reduced while maintaining the necessary mechanical behavior by substituting the 3D corrugated cardboard core structure with an equivalent 2D model. Homogenization modeling was then employed to develop a procedure for determining the fatigue strength of cardboard boxes. Numerical simulations of the finite element model were performed under cyclic loading conditions that mirror those in actual cargo transportation. The fatigue cycles obtained were validated through comparison with experimental data. The results demonstrate that the proposed homogenization model and fatigue strength determination procedure provide good accuracy, efficiency, and suitability for fatigue analysis in the packaging optimization design process.

ỨNG DỤNG MÔ HÌNH ĐỒNG NHẤT HÓA ĐỂ XÁC ĐỊNH ĐỘ BỀN MỎI CỦA BAO BÌ CARTON

Lương Việt Dũng
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THÔNG TIN BÀI BÁO

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TỪ KHÓA

Độ bền mỏi
Carton lõi sóng
Tải chu kỳ
Đồng nhất hóa
Phần tử hữu hạn

TÓM TẮT

Bao bì bằng carton thường được sử dụng trong vận chuyển, phân phối và lưu trữ. Nó thường phải chịu tác động cơ học lặp đi lặp lại dẫn đến việc phá hỏng bao bì. Bài báo này trình bày một phương pháp tiếp cận số dựa trên việc sử dụng mô hình đồng nhất hóa để xây mô hình phần tử hữu hạn, từ đó xác định độ bền mỏi của carton lõi lượn sóng. Bằng cách thay thế cấu trúc 3D tấm carton lõi lượn sóng bằng mô hình 2D tương đương đã giúp giảm đáng kể chi phí tính toán trong khi vẫn bảo toàn các ứng xử cơ học cần thiết. Tiếp đó mô hình đồng nhất hóa được sử dụng để xây dựng quy trình xác định độ bền mỏi cho hộp carton. Các mô phỏng số cho mô hình phần tử hữu hạn đã được tiến hành trong điều kiện tải chu kỳ giống như trong thực tế vận chuyển hàng hóa. Chu kỳ mỏi xác định được được xác thực bằng cách so sánh với dữ liệu thực nghiệm. Kết quả chỉ rằng mô hình đồng nhất hóa và quy trình xác định độ bền mỏi đề xuất cho kết quả có độ chính xác, hiệu quả tốt và phù hợp để phân tích mỏi trong quy trình thiết kế tối ưu hóa bao bì.

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1. Introduction

During handling, stacking, and transportation of goods, packaging is often subjected to repeated mechanical loads. These cyclic loads can cause fatigue damage, thereby reducing the structural integrity and protective function of the packaging. Packaging materials and structures also require sustainable and lightweight materials. Corrugated board has emerged as the first choice due to its recyclability, low production costs, and satisfactory mechanical performance. However, the complex structure consisting of flat sheets and corrugated cores poses significant challenges in accurately predicting its mechanical behavior, especially under fatigue loading conditions. To overcome these limitations, corrugated board studies have used homogenization techniques to represent the overall behavior of corrugated core boards by converting them into equivalent homogeneous flat boards. This equivalent model allows efficient simulation of large-scale packaging systems without significantly reducing the accuracy of stress and strain predictions. Previous studies have addressed many aspects of corrugated core board modeling. Linear and orthotropic elastic models have been widely used to predict static loads [1]-[2]. Recent studies have also addressed the homogeneous model describing the plastic behavior to improve the accuracy and more fully describe the behavior of the board [3]–[9].

To date, many studies related to the determination of the fatigue strength of cardboard have been published [10]–[13]. The study results in the prediction of the fatigue strength of cardboard packaging under real transportation conditions by combining static and dynamic analysis using the finite element method (FEM). After each vibration cycle, the compressive strength of the box was re-evaluated to determine the degree of degradation due to dynamic loading. The results showed that the compressive strength of the packaging decreased gradually after each impact, with the decrease depending on the size and shape of the box [14]. The study to determine the fatigue strength of packaging during transport was carried out by measuring and analyzing the vibration and impact levels that bottled beer is subjected to when transported by truck on the Belgian road network. The results showed that low-frequency vibrations (5–25Hz) increased significantly as the stacking height of the cartons increased, especially in plastic cartons. Vibrations above 25 Hz could be amplified up to 9 times depending on the stacking height of the plastic cartons. At the same time, cartons were able to dampen vibrations above 25 Hz and absorb shocks better [15]. The influence of climatic conditions on the static and dynamic strength of carton boxes was carried out in the study of Lamb et al. [12]. The study showed that static tests are not sufficient to evaluate the fatigue resistance of paper packaging under changing climatic conditions, and it is necessary to combine dynamic tests to ensure durability and safety during transportation. Using FEM, the study simulated the mechanical properties of corrugated cardboard boxes, including compression, bending and shear stress. The simulation results were compared with experimental data to confirm the accuracy of the model [16]. A method for evaluating the strength of corrugated core cardboard is proposed by proposing new tensile and shear tests, taking into account the adhesive bond strength between layers. The results showed that the new method helps to obtain reliable, reproducible strength values, while also allowing for to evaluation of the influence of the environment and layer structure on the tensile and shear strength [17]. The mechanical behavior of corrugated cardboard boxes under the impact of loads was studied to understand the load-bearing and deformation capacity of this material under actual usage conditions. The study simulated the mechanical properties of corrugated cardboard boxes, including compression, bending, and shear stress. The simulation results were compared with experimental data to confirm the accuracy of the model. The stiffness of the corrugated cardboard core was also studied through the deformation of the sheet in three regions: elastic, transition, and plastic [18].

In most publications related to corrugated sandwich structures, there is no study using the homogenization model to determine the fatigue strength of packaging, especially carton packaging. This study applies the homogenization model to determine the fatigue strength of the corrugated cardboard sheet. From there, a process for determining the fatigue strength of corrugated cardboard packaging is developed.

2. Theoretical foundations

2.1. Homogenization model

The homogenization method for corrugated cardboard panels is performed with a representative element, as shown in Figure 1. The relationship between the size components is expressed in equation 1. The size of this periodic unit cell is ideally sufficient to represent the panel and is comparable to the entire panel. The homogenization method utilizes the integration method. For each paper layer, three integration points in the thickness direction are defined. The plasticity algorithm is applied at each integration point to determine the stress state and the elastic-plastic matrix in the local coordinate system of each layer [1]–[9]. The structure of the 3D carton board is converted into an equivalent 2D board through the calculation of the equivalent stiffness. The membrane force, bending, and torsional moment are obtained by integrating the stress over the thickness to derive the overall stiffness matrix [9]:

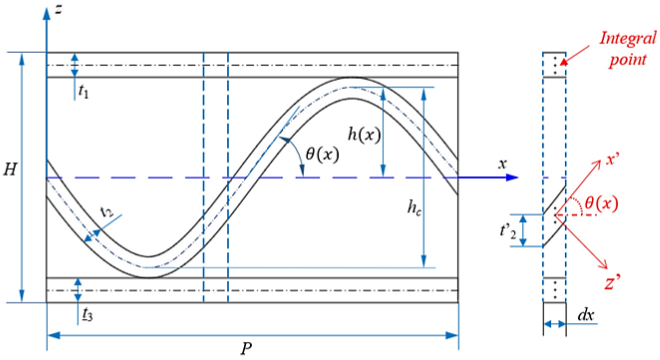

Figure 1. *Periodic unit cell of a corrugated cardboard [9]*

$$h(x) = \frac{h_c}{2}\sin(\frac{2\pi x}{P})\,; \quad \theta(x) = \arctan(\frac{dh(x)}{dx}) \tag{1}$$

$$A_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)} w_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}\left(\theta(x)\right) w_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)} w_k \tag{2}$$

$$B_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)} z_k w_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}\left(\theta(x)\right) z_k w_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)} z_k w_k \tag{3}$$

$$D_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)} z_k^2 w_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}\left(\theta(x)\right) z_k^2 w_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)} z_k^2 w_k \tag{4}$$

2.2. Fatigue Criterion

In this study, the Fe-Safe software was utilized to obtain an initial numerical approach to the fatigue behavior of cardboard packaging. The existing model in Fe-Safe was employed and will be described in this section [19]. Stress life generally refers to the number of cycles to failure of structures subjected to alternating and repeated loading, resulting in the initiation and development of cracks, and ultimately damage to the structure. To investigate the cyclic behavior of a material, a specimen is typically cycled between fixed strain limits, and the stress response may indicate that the material softens, producing lower stresses for each application of strain. After a certain number of cycles, the cyclic stress-strain curve stabilizes, resulting in a stable hysteresis loop. For other materials, the stress response may indicate that the material hardens. The material life equation can be decomposed into an elastic and a plastic component as follows:

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E}(2N_f)^b \quad (5)$$

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f(2N_f)^c \quad (6)$$

Where b is the fatigue strength exponent, c is the fatigue ductility exponent, ε'_f is the fatigue ductility coefficient, σ'_f is the fatigue strength coefficient, and N_f is the number of cycles.

The relationship between actual local stress amplitude and strength, and between actual local strain amplitude and strength is given by equations (7) and (8):

$$\frac{\Delta\sigma}{2} = \sigma'_f(2N_f)^b \quad (7)$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \quad (8)$$

3. Results and Discussion

3.1. Numerical simulation of fatigue strength of cardboard panels

Experimental determination of bending fatigue strength of corrugated core cardboard panels was carried out in the study of Wang et al. [20]. Accordingly, the material used in the study is a type B carton. The results of three-point bending experiments determined the fatigue cycle of each test sample corresponding to different load levels. The homogeneous model was used to simulate the three-point static bending experiment in the study of Luong et al. [8]. For the three-point bending test, bending stress and strain are calculated by the following equations:

$$\sigma = \frac{3PL}{2bd^2} \quad (9)$$

$$\varepsilon = \frac{6Dd}{L^2} \quad (10)$$

Where σ and ε are the stress and strain in the middle of the sample on the outer fiber, respectively, P is the applied load, D is the deflection in the middle of the sample, L, b and d are the distance between the fixed supports, the width and thickness of the sample, respectively.

The results obtained are shown in Figure 2. It is easy to see that the results of the 2D homogeneous model are in good agreement with the experimental results and the 3D model. However, the calculation time of the 2D model is significantly reduced. Therefore, this study uses the homogeneous model to simulate the number of cyclically loaded carton panels in Abaqus software. Then, the fatigue strength of the panels is determined through the Fe-Safe fatigue analysis software.

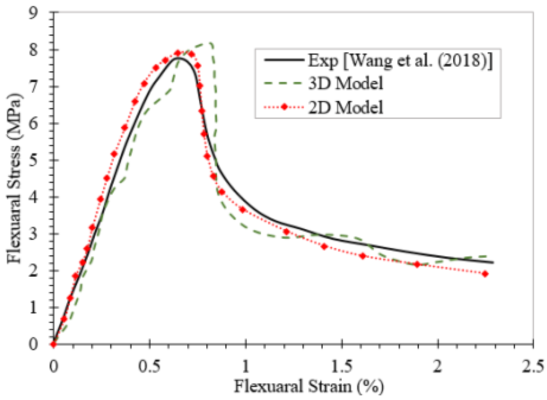

Figure 2. *Comparison of bending stress-strain [8]*

First, a bending fatigue test simulation with the same conditions as the experiment by Wang and Sun [20] was performed. The length of the specimen (in the CD direction) is 140 mm, and the width (in the MD direction) is 40 mm. The 2D model of the homogeneous cardboard sheet is composed of 1330 S4R quadrilateral elements and 1420 nodes. The specimen is placed on two rigid supports with a radius of 10 mm. The load is applied to the middle of the specimen by another rigid support with the same radius as the previous supports. The boundary conditions of the three-point bending fatigue test simulation are shown in Figure 3. The fatigue parameters of corrugated core cardboard sheet material were determined as shown in Table 1.

Table 1. *Fatigue properties of corrugated cardboard according to Wang and Sun*

Fatigue ductility coefficient	Fatigue ductility exponent	Fatigue strength coefficient (MPa)	Fatigue strength exponent	Cyclic strength coefficient (MPa)	Cyclic hardening exponent
0.035	-0.189	7.85	-0.0139	12.37	0.0743

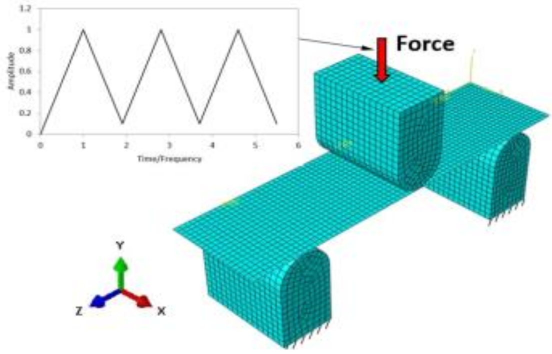

Figure 3. *Boundary conditions for the three-point bending fatigue test*

To determine the fatigue life of the specimens by numerical simulation, we combined the Abaqus finite element simulation software with the Fe-Safe fatigue calculation software. First, numerical simulations were performed with a cyclic load of constant sinusoidal amplitude applied at a frequency of 5 Hz and a stress ratio of 0.1. The stress level used was 0.90 of the ultimate stress (σ = 7.47 MPa). The ODB result file from each simulation was then imported into the Fe-Safe software to determine the number of fatigue cycles. The predicted fatigue life results for the specimens are given in Figure 4. The minimum life is at the bending load application, where the stress is maximum. This result is in good agreement with the experimental results, and the fatigue cycle error between simulation and experiment is 2.4% (Table 2). The results show that the homogenization model can be used to determine fatigue strength in carton packaging design.

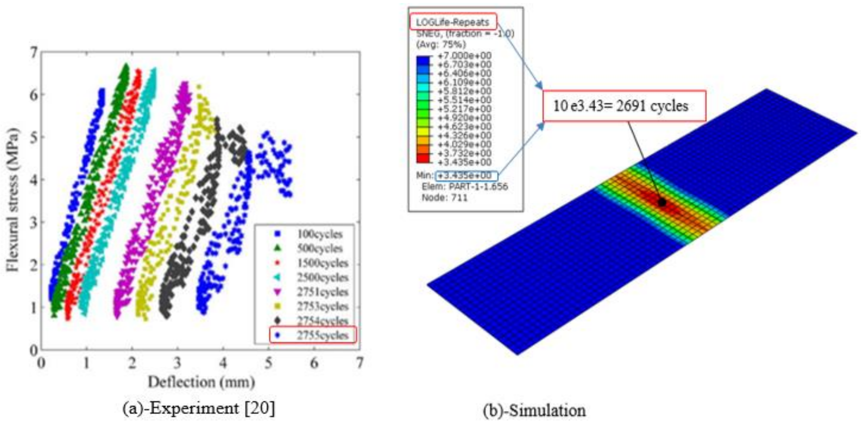

Figure 4. *Life of specimens at different stress levels*

Table 2. *Comparison of fatigue cycles determined by experiment and numerical simulation*

	Experiment [22]	Simulation	Error
Fatigue cycle	2758	2691	2.4 %

3.2. Simulation of the fatigue cycle of a carton box

In the study of Luong et al. [6], the experiments were conducted on a type B cardboard box subjected to vibration on a vibrating table. The carton box has dimensions of height 180 mm, length 300 mm and width 200 mm. The number of impacts, with a constant excitation acceleration amplitude, of the box was calculated as the number of fatigue cycles. The experimental setup and experimental modeling are shown in Figure 5. The homogenization model was applied to calculate the fatigue strength of a carton box. However, the data on the material parameters for fatigue of the carton board in this study were not complete, so the data in Table 1 were used. The procedure for determining the fatigue strength of the box was performed as follows:

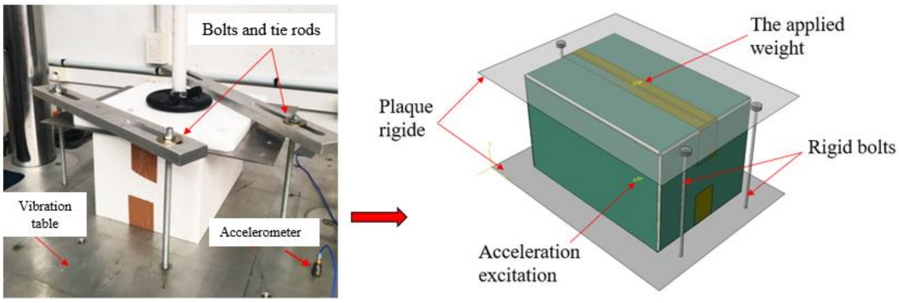

Figure 5. *Modeling of the shock test on a vibration table [6]*

➢ Experimental numerical simulations were performed in Abaqus software to obtain the ODB result file. Note, to improve the accuracy of the numerical simulation results, the cardboard box after simulation will have the parts at the folding position removed, as shown in Figure 6. The reason is that during the folding process to create the cardboard box, the cardboard sheet was damaged at these positions.

➢ The ODB file was imported into the Fe-Safe software, and the fatigue parameters of the material were also entered. The result obtained is the fatigue strength of the carton box.

The results obtained are fatigue cycles or number of impacts shown in Figure 7 corresponding to the life of the carton box for impacts with increasing excitation acceleration amplitude. These results show that as the severity of the impact increases, the number of cycles required to cause damage to the box is consistent with experimental observations. The results also show that the procedure for determining the fatigue strength of carton packaging by combining Abaqus and Fe-Safe software is reliable (where g is the acceleration due to gravity).

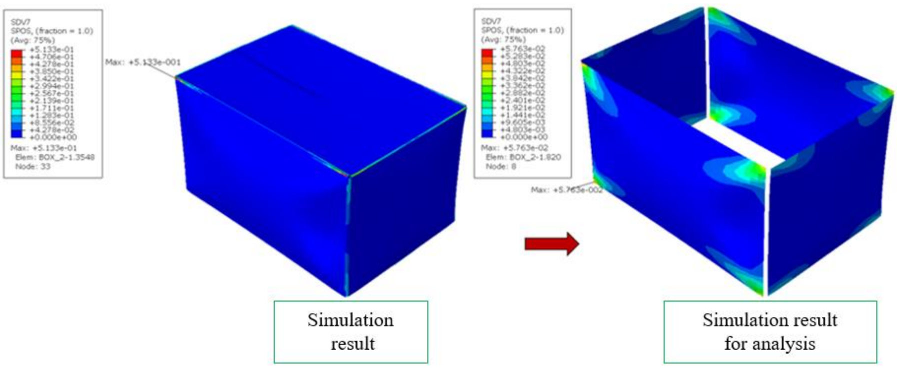

Figure 6. *Model selected for fatigue analysis*

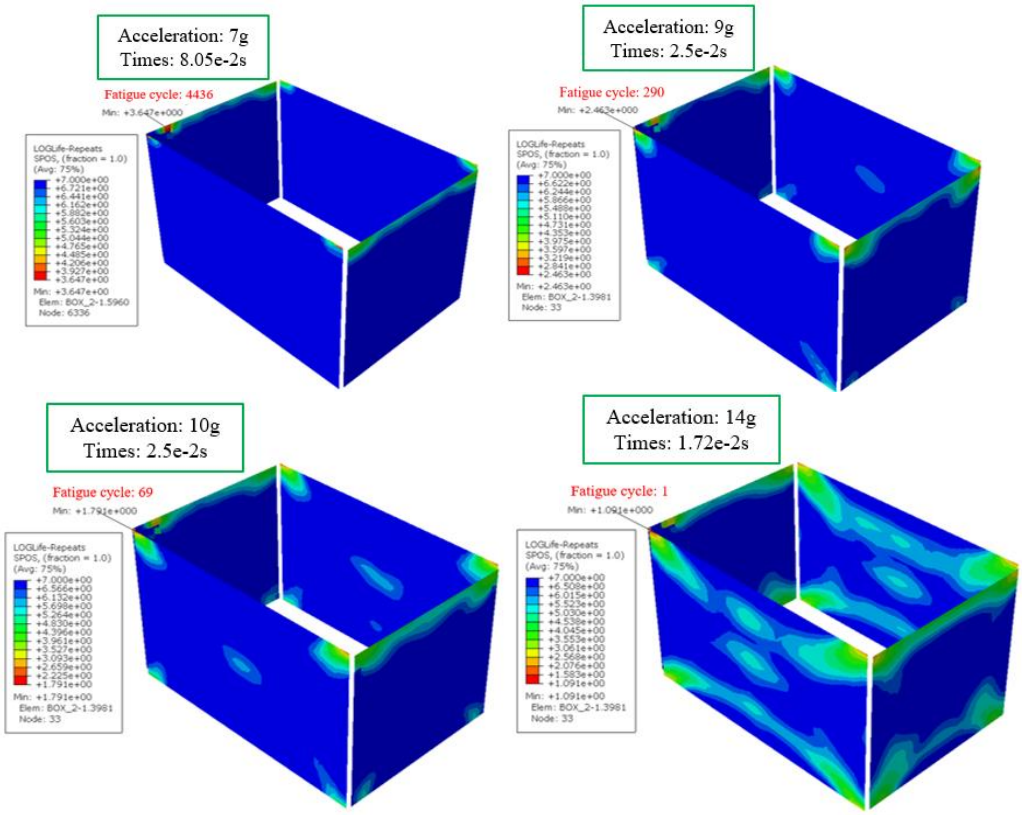

Figure 7. *Carton box life at different impact levels*

4. Conclusion

This study applied a homogenization model to determine the fatigue strength of corrugated core panels under cyclic loading. By replacing the 3D structure of the panel with an equivalent homogenized 2D panel. The 2D homogenization model proved to be sufficiently accurate in predicting the fatigue behavior. The homogenization model for fatigue studies was validated against experimental data, showing good agreement and confirming its potential as an efficient and reliable approach for evaluating structures under cyclic loading. The homogenization model proved to be an effective tool for simulating the fatigue behavior of corrugated core panels. It provides a good balance between accuracy and computational time, allowing its use in packaging design and durability assessment. The homogenization model combined with the proposed fatigue strength determination procedure is the basis for taking into account the effects of temperature and dynamic loading conditions commonly encountered in real packaging applications.

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