FORMATION TRACKING CONTROL OF OCEAN VEHICLES

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ABSTRACT

We present an application of our constructive method for design cooperative controllers in (Khac-Duc Do et al., 2018) to solve the problem of forcing a group of N ocean vehicles under environmental disturbances to track desired paths in a horizontal plane. The reader is referred to (Khac-Duc Do et al., 2018) for a survey of the formation control field.

Keywords: *Formation tracking control, ocean vehicles.*

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ĐIỀU KHIỂN BÁM NHÓM CÁC PHƯƠNG TIỆN GIAO THÔNG ĐƯỜNG BIỂN

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TÓM TẮT

Trình bày một ứng dụng lý thuyết điều khiển nhóm trong (Khac-Duc Do và cộng sự, 2019) vào việc thiết kế các bộ điều khiển nhóm bám trong mặt phẳng nằm ngang cho một nhóm N phương tiện đường biển chịu tác động của môi trường biển. Người đọc tham khảo (Khac-Du Do và cộng sự, 2019) để khảo sát về lý thuyết thiết kế điều khiển hợp tác.

Từ khóa: *Điều khiển nhóm, các phương tiện giao thông đường biển.*

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MATHEMATICAL MODEL AND CONTROL OBJECTIVE

The equations of motion of the i^{th} ocean vehicle such as surface ships and underwater vehicles moving in a horizontal plane (for clarity roll, pitch and heave motions are ignored) can be written as 0:

$$\begin{aligned} &\dot{\xi}_i = J(\psi_i)\vartheta_i \\ &M_i\dot{\vartheta}_i = -C_i(\vartheta_i)\vartheta_i - (D_i + D_{in}(\vartheta_i))\vartheta_i + \tau_i + J^T(\psi_i)b_i \end{aligned} \tag{1}$$

with $\xi_i = [x_i\ y_i\ \psi_i]^T$, $\vartheta_i = [u_i\ v_i\ r_i]^T$, $\tau_i = [\tau_{ui}\ \tau_{vi}\ \tau_{ri}]^T$, $b_i = [b_{ui}\ b_{vi}\ b_{ri}]^T$,

$$J(\psi_i) = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) & 0 \\ \sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}, M_i = \begin{bmatrix} m_{i11} & 0 & 0 \\ 0 & m_{i22} & m_{i23} \\ 0 & m_{i32} & m_{i33} \end{bmatrix}, C_i(\vartheta_i) = \begin{bmatrix} 0 & 0 & C_{i13} \\ 0 & 0 & C_{i23} \\ C_{i31} & C_{i32} & 0 \end{bmatrix}, D_i + D_{in}(\vartheta_i) = \begin{bmatrix} D_{i11} & 0 & 0 \\ 0 & D_{i22} & D_{i23} \\ 0 & D_{i32} & D_{i33} \end{bmatrix} \tag{2}$$

where

$$\begin{aligned} &m_{i11} = m_i - X_{i\dot{u}}, m_{i22} = m_i - Y_{i\dot{v}}, m_{i23} = m_i x_{ig} - Y_{i\dot{r}}, m_{i32} = m_i x_{ig} - N_{i\dot{v}}, m_{i33} = I_{iz} - N_{i\dot{r}}, \\ &C_{i13} = -C_{i31} = -m_{i22}v_i - 0.5(m_{i23} + m_{i32})r_i, C_{i23} = -C_{i32} = m_{i11}u_i, D_{i11} = -(X_{iu} + X_{i|u|u}|u_i|), \\ &D_{i22} = -(Y_{iv} + Y_{i|v|v}|v_i| + Y_{i|r|v}|r_i|), D_{i23} = -(Y_{ir} + Y_{i|v|r}|v_i|), D_{i32} = -(N_{iv} + N_{i|v|v}|v_i| + N_{i|r|v}|r_i|), \\ &D_{i33} = -(N_{ir} + N_{i|v|r}|v_i| + N_{i|r|r}|r_i|) \end{aligned} \tag{3}$$

where x_i, y_i are the surge and sway displacements, ψ_i is the yaw angle with coordinates in the earth fixed frame; u_i, v_i and r_i denote surge, sway and yaw velocities with coordinates in the body-fixed frame; m_i is the mass of the ship; I_{iz} is the ship's inertia about the Z_{ib}-axis of the body-fixed frame; x_{ig} is the X_{ib}-coordinate of the ship center of gravity, O_{ic}, in the body-fixed frame (see Figure 1); the controls τ_{iu}, τ_{iv} and τ_{ir} are the surge and sway forces and yaw moment in the body-fixed frame; b_{iu}, b_{iv} and b_{ir} are the constant disturbance forces and moment acting on surge, sway and yaw axes. The other symbols are referred to as hydrodynamic derivatives 0. For example, the hydrodynamic added mass force Y_i along the y_i-axis due to an acceleration $\dot{u}_i$ in the x_i-direction is written as $Y_i = -Y_{i\dot{u}}\dot{u}_i$ with $Y_{i\dot{u}} := \partial Y_i / \partial \dot{u}_i$. We assume that all the ship parameters and disturbances are unknown but constant.

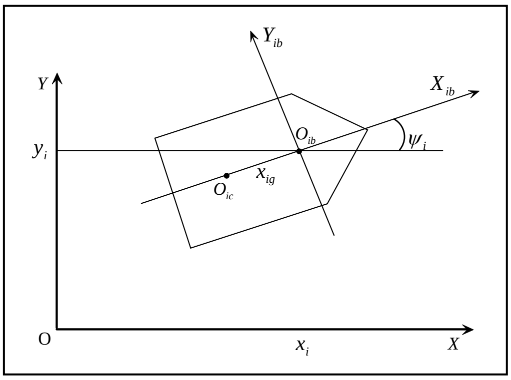

Figure 1. *Vessel coordinates*

Since collision is related to the position (x_i, y_i) of the vessel, we decouple the model (1) into the "position" and "orientation" models as follows

$$\begin{cases} \dot{q}_i = \chi_i \\ \dot{\chi}_i = \begin{bmatrix} \bar{\tau}_{ui} \\ \bar{\tau}_{vi} \end{bmatrix} + \begin{bmatrix} \Upsilon_{ui} \\ \Upsilon_{vi} \end{bmatrix} b_i \end{cases} \qquad \begin{cases} \dot{\psi}_i = r_i \\ \dot{r}_i = \bar{\tau}_{ri} + \Upsilon_{ri} b_i \end{cases} \tag{4}$$

where

$$q_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \chi_i = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) \\ \sin(\psi_i) & \cos(\psi_i) \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \tag{5}$$

and we have chosen the control τ_i as

$$\tau_i = C_i(\vartheta_i)\vartheta_i + (D_i + D_{in}(\vartheta_i))\vartheta_i - M_i J^{-1}(\psi_i)\left\{\dot{J}(\psi_i)\vartheta_i - [\bar{\tau}_{ui}\ \bar{\tau}_{vi}\ \bar{\tau}_{ri}]^T\right\} \tag{6}$$

where $\bar{\tau}_{ui}, \bar{\tau}_{vi}$ and $\bar{\tau}_{ri}$ are new controls to be designed; $\Upsilon_{ui}, \Upsilon_{vi}$ and Υ_{ri} are the first, second and third rows of $J(\psi_i)M_i^{-1}J^T(\psi_i)$, i.e.

$$\left[\Upsilon_{ui}\ \Upsilon_{vi}\ \Upsilon_{ri}\right]^T = J(\psi_i)M_i^{-1}J^T(\psi_i). \quad (7)$$

In this section, we consider the problem of designing the control input τ_i or $(\bar{\tau}_{ui}, \bar{\tau}_{vi}, \bar{\tau}_{ri})$ for each vehicle i that forces the group of N vehicles whose dynamics are given in (1) or (4) to track a moving changeable desired formation graph in the sense that the desired formation graph is allowed to move on a desired trajectory $\mathbf{\Gamma_{od}}$, and is allowed to change its shape including rotation, contraction and expansion, see Figure 2. The group of N vessels needs N individual reference trajectories. The desired formation is achieved by forcing each vessel to track its reference trajectory. We consider the formation graph whose center $\hat{O}$ moves along a reference trajectory $\mathbf{\Gamma_{od}}(s)$ with s being the path parameter. We assume that $\mathbf{\Gamma_{od}}(s)$ is regular in the sense that it is single valued and its first and second derivatives exist and are bounded. Since the formation graph under consideration is only representative, the center does not have to be the center of the graph but can be any convenient point. The shape of the graph can be varied by specifying the coordinates as a function of $\eta \in \mathbb{R}^m$ called the formation shape parameter vector, from each vertex i to the center of the graph. The parameter vector η is used to specify rotation, expansion and contraction of the formation such that when η converges to its desired value η_f, the desired shape of the formation is achieved. When the graph moves along the trajectory $\mathbf{\Gamma_{od}}(s)$, the vertex i generates the reference trajectory $q_{id}(s,\eta)$ for the agent i. Designing the control input τ_i or $(\bar{\tau}_{ui}, \bar{\tau}_{vi}, \bar{\tau}_{ri})$ for each agent i that directly forces the vessel i to track its reference trajectory $q_{id}(s,\eta)$ is difficult except for the case where the trajectory $q_{id}(s,\eta)$ is a straight line due to collision avoidance taken into account. Therefore we consider the dynamics of the vessels in the moving coordinate frame attached to the graph and its origin coincides with the center of the graph.

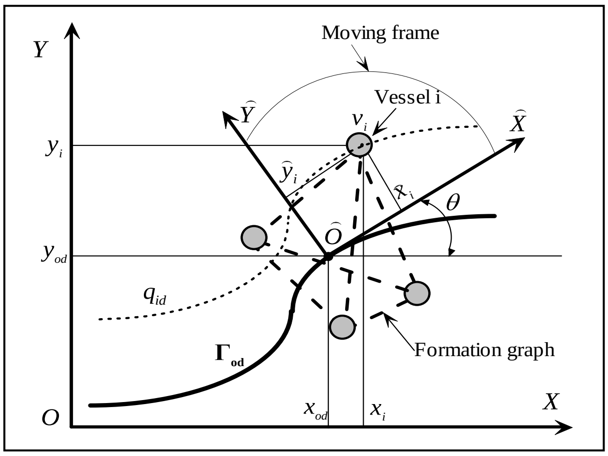

Figure 2. *Formation coordinates in 2D.*

The control objective is formally stated as follows:

Control objective: Assume that at the initial time t_0 each vessel starts from a different location, and that each vessel has a different desired location on its reference trajectory $q_{id}(s,\eta)$, i.e. there exists a strictly positive constant d_{ij}, which is referred to as the minimum safe distance between the vessel i and the vessel j, such that

$$\begin{aligned} &\| q_i(t_0) - q_j(t_0) \| \geq d_{ij} \\ &\| q_{id} - q_{jd} \| \geq d_{ij},\ \forall i, j \in \{1, 2, \ldots N\}. \end{aligned} \quad (8)$$

Design the control input $(\bar{\tau}_{ui}, \bar{\tau}_{vi}, \bar{\tau}_{ri})$ for each vessel i and an update law for the unknown disturbance vector b_i, and the formation vector η such that position and yaw angle of each vessel (almost) globally asymptotically tracks its reference trajectory $q_{id}(s,\eta)$ and ψ_{id}, while avoids collisions with all other vessels in the group, i.e.

$$\begin{aligned} &\lim_{t\to\infty}(q_i(t) - q_{id}) = 0 \\ &\lim_{t\to\infty}(\psi_i(t) - \psi_{id}) = 0 \\ &\| q_i(t) - q_j(t) \| \geq d_{ij},\ \forall i, j \in \{1, 2, \ldots N\},\ \forall t \geq t_0 \geq 0 \\ &\lim_{t\to\infty}(\eta(t) - \eta_f) = 0. \end{aligned} \quad (9)$$

CONTROL DESIGN

As mentioned before, we now consider the dynamics of the agents in the moving coordinate frame, $\widehat{O}\widehat{X}\widehat{Y}$ attached to the formation graph, see Figure 6. The origin $\widehat{O}$ of this frame coincides with the center of the graph, and is on the reference trajectory $\mathbf{\Gamma_{od}}(x_{od}(s), y_{od}(s))$. The $\widehat{O}\widehat{X}$ and $\widehat{O}\widehat{Y}$ axes of this frame are tangential and perpendicular to the reference trajectory $\mathbf{\Gamma_{od}}(x_{od}(s), y_{od}(s))$. Therefore the angle θ between the $\widehat{O}\widehat{X}$ and OX is calculated as $\theta = \arctan(y_d^{'} / x_d^{'})$, where $\bullet' \triangleq \partial \bullet / \partial s$. Let the coordinates and desired coordinates of the agent i assigned to the vertex i of the formation graph in the moving frame $\widehat{O}\widehat{X}\widehat{Y}$ be $\widehat{q}_i = (\widehat{x}_i, \widehat{y}_i)$ and $\widehat{q}_{id}(\eta) = (\widehat{x}_{id}(\eta), \widehat{y}_{id}(\eta))$. Therefore, if we are able to design the control input $(\overline{\tau}_{ui}, \overline{\tau}_{vi}, \overline{\tau}_{ri})$ for the agent i such that

$$\begin{aligned} &\lim_{t\to\infty}(\widehat{q}_i(t) - \widehat{q}_{id}(\eta)) = 0, \\ &\lim_{t\to\infty}(\eta(t) - \eta_f) = 0, \\ &\| \widehat{q}_i(t) - \widehat{q}_i(t) \| \geq d_{ij}, \\ &\lim_{t\to\infty}(\psi_i(t) - \psi_{id}) = 0,\ \forall t \geq t_0 \geq 0 \end{aligned} \tag{10}$$

where ψ_{id} is the desired yaw angle of the vessel i, and let the moving frame $\widehat{O}\widehat{X}\widehat{Y}$ moves along the trajectory $\mathbf{\Gamma_{od}}(x_{od}(s), y_{od}(s))$, then the control objective is solved. A simple choice of the desired angle is $\psi_{id} = \theta$. This choice implies that we want the yaw angle ψ_i of all vessels to approach the same value $\theta = \arctan(y_d^{'} / x_d^{'})$. From Figure 6, we have

$$\widehat{q}_i = R(\theta)(q_i - q_{od}) \tag{11}$$

where $q_{od} = [x_{od}\ \ y_{od}]^T$ and $R(\theta)$ is the rotation matrix given by

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \tag{12}$$

It is noted that $R(\theta)$ is indeed invertible for all $\theta \in \mathbb{R}$. Differentiating both sides of (11) along the solutions of (4) gives

$$\begin{aligned} &\begin{cases} \dot{\widehat{q}}_i = \widehat{\chi}_i \\ \dot{\widehat{\chi}}_i = \ddot{R}(\theta)(q_i - q_{od}) + 2\dot{R}(\theta)(\chi_i - \dot{q}_{od}) + R(\theta)\left(\begin{bmatrix} \overline{\tau}_{ui} \\ \overline{\tau}_{vi} \end{bmatrix} + \begin{bmatrix} \Upsilon_{ui} \\ \Upsilon_{vi} \end{bmatrix} b_i - \ddot{q}_{od}\right) \end{cases} \\ &\begin{cases} \dot{\psi}_i = r_i \\ \dot{r}_i = \overline{\tau}_{ri} + \Upsilon_{ri} b_i \end{cases} \end{aligned} \tag{13}$$

where $\widehat{\chi}_i = R(\theta)(\chi_i - \dot{q}_{od}) + \dot{R}(\theta)(q_i - q_{od})$. Since the system (13) is of a strict feedback form, we will use the backstepping technique and the technique developed in the previous section to design the control $(\overline{\tau}_{ui}, \overline{\tau}_{vi}, \overline{\tau}_{ri})$ to achieve the control objective. The control design consists of two steps as follows.

Step 1. At this step, we consider $\widehat{\chi}_i$ and r_i as controls. Define

$$\begin{aligned} \tilde{\widehat{\chi}}_i &= \widehat{\chi}_i - \alpha_{\widehat{\chi}_i} \\ \tilde{r}_i &= r_i - \alpha_{r_i} \end{aligned} \tag{14}$$

where $\alpha_{\widehat{\chi}_i}$ and α_{r_i} are virtual controls of $\widehat{\chi}_i$ and r_i, respectively. In order to design $\alpha_{\widehat{\chi}_i}$ and α_{r_i}, we consider the following potential function:

$$\widehat{\varphi}_i = \widehat{\gamma}_i + \delta\widehat{\beta}_i + 0.5\psi_{ie}^2 + 0.5\| \eta - \eta_f \|^2 \tag{15}$$

where δ is a positive tuning constant, and $\psi_{ie} = \psi_i - \psi_{id}$. The functions $\hat{\gamma}_i$ and $\hat{\beta}_i$ are the goal and related collision avoidance functions specified as follows (see Subsection 3.2 for motivation):

$$\hat{\gamma}_i = \frac{1}{2}\| \hat{q}_i - \hat{q}_{id} \|^2, \ \hat{\beta}_i = \sum_{j \in N_i} \left(\frac{\hat{\beta}_{ij}^k}{\hat{\beta}_{ijd}^{2k}} + \frac{1}{\hat{\beta}_{ij}^k} \right) \tag{16}$$

where N_i is the set of the agents which are adjacent to the agent i and

$$\hat{\beta}_{ij} = \frac{1}{2}(\| \hat{q}_i - \hat{q}_j \|^2 - d_{ij}^2), \ \hat{\beta}_{ijd} = \frac{1}{2}(\| \hat{q}_{id} - \hat{q}_{jd} \|^2 - d_{ij}^2). \tag{17}$$

It is noted the yaw angle is not included in the collision avoidance function $\hat{\beta}_{ij}$ since it does not contribute to collisions. Differentiating both sides of (15) along the solutions of (13) with the use of (14), (16) and (17) gives

$$\dot{\hat{\varphi}}_i = \hat{\Omega}_i^T(\alpha_{\hat{\chi}_i} + \tilde{\hat{\chi}}_i) - \sum_{j \in N_i} \hat{\Phi}_{ij}^T(\alpha_{\hat{\chi}_j} + \tilde{\hat{\chi}}_j) + \psi_{ie}(\alpha_{r_i} + \tilde{r}_i - \dot{\psi}_{id}) - \hat{\Lambda}_i^T \dot{\eta} + (\eta - \eta_f)^T \dot{\eta} \tag{18}$$

$$\hat{\Phi}_{ij} = \delta k \left(\frac{1}{\hat{\beta}_{ijd}^{2k}} - \frac{1}{\hat{\beta}_{ij}^{2k}} \right) \hat{\beta}_{ij}^{k-1}(\hat{q}_i - \hat{q}_j)$$

where $\hat{\Omega}_i = \hat{q}_i - \hat{q}_{if} + \sum_{j \in N_i} \hat{\Phi}_{ij}$ (19)

$$\hat{\Lambda}_i = \left[(\hat{q}_i - \hat{q}_{id})^T \frac{\partial \hat{q}_{id}}{\partial \eta} + 2\delta k \sum_{j \in N_i} \frac{\hat{\beta}_{ij}^k}{\hat{\beta}_{ijd}^{2k+1}} (\hat{q}_{id} - \hat{q}_{jd})^T \left(\frac{\partial \hat{q}_{id}}{\partial \eta} - \frac{\partial \hat{q}_{jd}}{\partial \eta} \right) \right]^T.$$

The equation (18) suggests that we choose the controls $\alpha_{\hat{\chi}_i}$ and α_{r_i} and the update $\dot{\eta}$ as

$$\begin{aligned} \alpha_{\hat{\chi}_i} &= -\hat{C}\hat{\Omega}_i \\ \alpha_{r_i} &= -\rho_i \psi_{ie} + \dot{\psi}_{id} \\ \dot{\eta} &= -\Gamma(\eta - \eta_f) \end{aligned} \tag{20}$$

where $\hat{C} \in \mathbb{R}_+^{2\times 2}$ and $\Gamma \in \mathbb{R}_+^{m\times m}$ are symmetric positive definite matrices, and ρ_i is a positive constant. Substituting (20) into (18) results in

$$\dot{\hat{\varphi}}_i = -\hat{\Omega}_i^T \hat{C} \hat{\Omega}_i - \rho_i \psi_{ie}^2 + \hat{\Omega}_i^T \tilde{\hat{\chi}}_i - \sum_{j \in N_i} \hat{\Phi}_{ij}^T(\alpha_{\hat{\chi}_j} + \tilde{\hat{\chi}}_j) + \psi_{ie}\tilde{r}_i + \hat{\Lambda}_i^T \Gamma(\eta - \eta_f) - (\eta - \eta_f)^T \Gamma(\eta - \eta_f). \tag{21}$$

Substituting (20) into the first two equations of (13) yields

$$\begin{aligned} \dot{\hat{q}}_i &= -\hat{C}\hat{\Omega}_i + \tilde{\hat{\chi}}_i \\ \dot{\psi}_{ie} &= -\rho_i \psi_{ie} + \tilde{r}_i. \end{aligned} \tag{22}$$

Step 2. At this step, we design the control τ_i and an update law for the disturbance vector b_i. Consider the following function

$$\hat{\omega}_i = \hat{\varphi}_i + 0.5\left(\tilde{\hat{\chi}}_i^T \tilde{\hat{\chi}}_i + \tilde{r}_i^2 + \tilde{b}_i^T \Gamma_{b_i}^{-1} \tilde{b}_i \right) \tag{23}$$

where $\tilde{b}_i = b_i - \hat{b}_i$ with $\hat{b}_i$ an estimate of b_i. Differentiating both sides of (23) the solutions of (21) and the last equation of (13) results in

$$\begin{aligned} \dot{\hat{\omega}}_i = & -\hat{\Omega}_i^T \hat{C} \hat{\Omega}_i - \rho_i \psi_{ie}^2 - \sum_{j \in N_i} \hat{\Phi}_{ij}^T(\alpha_{\hat{\chi}_j} + \tilde{\hat{\chi}}_j) + \hat{\Lambda}_i^T \Gamma(\eta - \eta_f) - (\eta - \eta_f)^T \Gamma(\eta - \eta_f) + \\ & \tilde{\hat{\chi}}_i^T \left(\ddot{R}(\theta)(q_i - q_{od}) + 2\dot{R}(\theta)(\chi_i - \dot{q}_{od}) + R(\theta)\left(\begin{bmatrix} \bar{\tau}_{ui} \\ \bar{\tau}_{vi} \end{bmatrix} + \begin{bmatrix} \Upsilon_{ui} \\ \Upsilon_{vi} \end{bmatrix} \hat{b}_i - \ddot{q}_{od} \right) + \hat{\Omega}_i - \dot{\alpha}_{\hat{\chi}_i} \right) + \\ & \tilde{r}_i \left(\bar{\tau}_{ri} + \Upsilon_{ri}\hat{b}_i + \psi_{ie} - \dot{\alpha}_{r_i} \right) + \left(\tilde{\hat{\chi}}_i^T R(\theta) \begin{bmatrix} \Upsilon_{ui} \\ \Upsilon_{vi} \end{bmatrix} + \tilde{r}_i \Upsilon_{ri} - \dot{\hat{b}}_i^T \Gamma_{b_i}^{-1} \right) \tilde{b}_i \end{aligned} \tag{24}$$

From (24), we choose the control τ_i and an update law for the unknown parameter vector Θ_i as follows:

$$\begin{bmatrix}\bar{\tau}_{ui}\\ \bar{\tau}_{vi}\end{bmatrix} = -\begin{bmatrix}\Upsilon_{ui}\\ \Upsilon_{vi}\end{bmatrix}\hat{b}_i + \ddot{q}_{od} - R^{-1}(\theta)\left(\dot{R}(\theta)(q_i - q_{od}) + 2\dot{R}(\theta)(\chi_i - \dot{q}_{od}) + \widehat{\Omega}_i - \dot{\alpha}_{\tilde{\chi}_i} + H_i\tilde{\tilde{\chi}}_i\right)$$
$$\bar{\tau}_{ri} = -\Upsilon_{ri}\hat{b}_i - \psi_{ie} + \dot{\alpha}_{r_i} - w_i\,\tilde{r}_i \qquad (25)$$
$$\dot{\hat{b}}_i = \Gamma_{b_i}\left(\tilde{\tilde{\chi}}_i^T R(\theta)\begin{bmatrix}\Upsilon_{ui}\\ \Upsilon_{vi}\end{bmatrix} + \tilde{r}_i\Upsilon_{ri}\right)^T$$

where H_i is a symmetric positive definite matrix, and w_i is a positive constant. Substituting (25) into (24) results in

$$\dot{\widehat{\omega}}_i = -\widehat{\Omega}_i^T\widehat{C}\widehat{\Omega}_i - \rho_i\psi_{ie}^2 - \tilde{\tilde{\chi}}_i^T H_i\tilde{\tilde{\chi}}_i - w_i\tilde{r}_i^2 - \sum_{j\in N_i}\widehat{\Phi}_{ij}^T(\alpha_{\tilde{\chi}_j} + \tilde{\tilde{\chi}}_j) + \widehat{\Lambda}_i^T\Gamma(\eta-\eta_f) - (\eta-\eta_f)^T\Gamma(\eta-\eta_f) \quad (26)$$

Indeed, substituting the control $(\bar{\tau}_{ui}, \bar{\tau}_{vi}, \bar{\tau}_{ri})$ into the derivative of $\tilde{\tilde{\chi}}_i$ and $\tilde{r}_i$ results in

$$\dot{\tilde{\tilde{\chi}}}_i = -H_i\tilde{\tilde{\chi}}_i + \begin{bmatrix}\Upsilon_{ui}\\ \Upsilon_{vi}\end{bmatrix}\tilde{b}_i - \widehat{\Omega}_i$$
$$\dot{\tilde{r}}_i = -w_i\tilde{r}_i - \psi_{ie} + \Upsilon_{ri}\tilde{b}_i. \qquad (27)$$

STABILITY ANALYSIS

We only show that with the control $(\bar{\tau}_{ui}, \bar{\tau}_{vi}, \bar{\tau}_{ri})$ and the update law for the disturbance vector given in (25), and the update law $\dot{\eta}$ for formation parameter (20), there are no collisions between agents, the solutions of the closed loop system consisting of (22), the third equations of (20) and (25), and (27) exist, and $\lim_{t\to\infty}\widehat{\Omega}_i = 0$. Proof of the critical point $\widehat{q} = \widehat{q}_d$ with $\widehat{q} = [\widehat{q}_1^T,...,\widehat{q}_N^T]$ and $\widehat{q}_d = [\widehat{q}_{1d}^T,...,\widehat{q}_{Nd}^T]$ being the asymptotically stable, and other equilibrium points being unstable or saddle follows the same lines as in Section 3. We consider the following function

$$\widehat{\omega}_{tot} = \log(1+\widehat{\omega}) + 0.5(\eta-\eta_f)^T(\eta-\eta_f) \qquad (28)$$

where

$$\begin{aligned}\widehat{\omega} &= \sum_{i=1}^N(\widehat{\omega}_i - 0.5\delta\widehat{\beta}_i + 0.5\psi_{ie}^2 + 0.5\tilde{\tilde{\chi}}_i^T\tilde{\tilde{\chi}}_i + 0.5\tilde{r}_i^2 + 0.5\tilde{b}_i^T\Gamma_{b_i}^{-1}\tilde{b}_i + 0.5(\eta-\eta_f)^T(\eta-\eta_f))\\ &= \sum_{i=1}^N(\widehat{\gamma}_i + 0.5\delta\widehat{\beta}_i + 0.5\psi_{ie}^2 + 0.5\tilde{\tilde{\chi}}_i^T\tilde{\tilde{\chi}}_i + 0.5\tilde{r}_i^2 + 0.5\tilde{b}_i^T\Gamma_{b_i}^{-1}\tilde{b}_i + 0.5(\eta-\eta_f)^T(\eta-\eta_f))\end{aligned} \qquad (29)$$

which is proper using the same arguments as in Subsection 3.2. Differentiating both sides of (28) along the solutions of (26) and the second equation of (20) satisfies

$$\dot{\widehat{\omega}}_{tot} = -\frac{1}{1+\widehat{\omega}}\sum_{i=1}^N(\widehat{\Omega}_i^T\widehat{C}\widehat{\Omega}_i + \rho_i\psi_{ie}^2 + \tilde{\tilde{\chi}}_i^T H_i\tilde{\tilde{\chi}}_i + w_i\tilde{r}_i^2) + \frac{1}{1+\widehat{\omega}}\sum_{i=1}^N\widehat{\Lambda}_i^T\Gamma(\eta-\eta_f) - \frac{2+\widehat{\omega}}{1+\widehat{\omega}}(\eta-\eta_f)^T\Gamma(\eta-\eta_f)$$

. (30)

From the expressions of $\widehat{\omega}$ and $\widehat{\Lambda}_i$, see (15), (16), (19) and (29), it can be checked that there exists a positive constant $\varepsilon_{\max}$ such that $\frac{1}{1+\widehat{\omega}}\sum_{i=1}^N\|\widehat{\Lambda}_i\| \le \varepsilon_{\max}$. (31)

Using (31), we can write (30) as

$$\dot{\widehat{\omega}}_{tot} \le \frac{\lambda_{\max}(\Gamma)}{4\varepsilon(1+\widehat{\omega})^2}\left(\sum_{i=1}^N\|\widehat{\Lambda}_i\|\right)^2 + \varepsilon\lambda_{\max}(\Gamma)\|\eta-\eta_f\|^2 - \lambda_{\min}(\Gamma)\|\eta-\eta_f\|^2 \qquad (32)$$

where ε is a positive constant, $\lambda_{\max}(\Gamma)$ and $\lambda_{\min}(\Gamma)$ denote the maximum and minimum eigenvalues of Γ. Picking $\varepsilon = \lambda_{\min}(\Gamma)/\lambda_{\max}(\Gamma)$, we can write (32) as

$$\dot{\hat{\omega}}_{tot} \le \frac{\lambda_{\max}^2(\Gamma)}{4\lambda_{\min}} \varepsilon_{\max}^2 \triangleq \bar{\varepsilon}_{\max} . \tag{33}$$

Integrating both sides of (33) from t_0 to t results in $\hat{\omega}_{tot}(t) \le \hat{\omega}_{tot}(t_0) + \bar{\varepsilon}_{\max}(t-t_0)$ (34)

From definition of $\hat{\omega}_{tot}$ we can write (34) as

$$\begin{aligned}&\sum_{i=1}^{N}(\hat{\gamma}_i(t)+0.5\delta\hat{\beta}_i(t)+0.5\psi_{ie}^2(t)+0.5\tilde{\bar{\chi}}_i^T(t)\tilde{\bar{\chi}}_i(t)+0.5\tilde{r}_i^2(t)+0.5\tilde{b}_i^T(t)\Gamma_{b_i}^{-1}\tilde{b}_i(t)+0.5(\eta(t)-\eta_f)^T(\eta(t)-\eta_f))\\ &\le \sum_{i=1}^{N}(\hat{\gamma}_i(t_0)+0.5\delta\hat{\beta}_i(t_0)+0.5\psi_{ie}^2(t_0)+0.5\tilde{\bar{\chi}}_i^T(t_0)\tilde{\bar{\chi}}_i(t_0)+0.5\tilde{r}_i^2(t_0)+0.5\tilde{b}_i^T(t_0)\Gamma_{b_i}^{-1}\tilde{b}_i(t_0)+\\ &0.5(\eta(t_0)-\eta_f)^T(\eta(t_0)-\eta_f))+\bar{\varepsilon}_{\max}(t-t_0)\end{aligned} \tag{35}$$

where

$$\begin{aligned}&\hat{\gamma}_i(t)=\frac{1}{2}\|\hat{q}_i(t)-\hat{q}_{id}\|^2,\ \hat{\beta}_i(t)=\sum_{j\in N_i}\left(\frac{\hat{\beta}_{ij}^k(t)}{\hat{\beta}_{ijd}^{2k}}+\frac{1}{\hat{\beta}_{ij}^k(t)}\right),\ \hat{\beta}_{ij}(t)=\frac{1}{2}\|\hat{q}_i(t)-\hat{q}_j(t)\|^2,\\ &\hat{\gamma}_i(t_0)=\frac{1}{2}\|\hat{q}_i(t_0)-\hat{q}_{id}\|^2,\ \hat{\beta}_i(t_0)=\sum_{j\in N_i}\left(\frac{\hat{\beta}_{ij}^k(t_0)}{\hat{\beta}_{ijd}^{2k}}+\frac{1}{\hat{\beta}_{ij}^k(t_0)}\right),\ \hat{\beta}_{ij}(t_0)=\frac{1}{2}\|\hat{q}_i(t_0)-\hat{q}_j(t_0)\|^2 .\end{aligned} \tag{36}$$

From (8) and (10) we have $\hat{\beta}_{ij}(t_0)$ and $\hat{\beta}_{ijd}$ are strictly larger than some positive constants. Therefore the right hand side of (35) cannot escape to infinity unless at the time $t=\infty$. Therefore, the left hand side of (35) cannot escape to infinity for all $t\in[t_0,\infty)$. This implies that $\beta_{ij}(t)$ cannot be equal to zero for all $t\in[t_0,\infty)$, i.e. no collisions can occur for all $t\in[t_0,\infty)$. Since the left hand side of (35) cannot be escape to infinity in a finite time, $q_i(t)$ cannot escape to infinity in a finite time. This means that the solutions of the closed loop system consisting of (22), the second equations of (20) and (25), and (27) exist. On the other hand, it is true from the second equation of (20) that $\|\eta(t)-\eta_f\|\le\|\eta(t_0)-\eta_f\|e^{-\lambda_{\min}(\Gamma)(t-t_0)}$ (37)

which implies that the desired formation shape is exponentially achieved. Substituting (37) into (30) yields $\dot{\hat{\omega}}_{tot} \le \lambda_{\max}(\Gamma)\varepsilon_{\max}\|\eta(t_0)-\eta_f\|e^{-\lambda_{\min}(t-t_0)}$. (38)

Integrating both sides of (38) from t_0 to t gives

$$\begin{aligned}&\sum_{i=1}^{N}(\hat{\gamma}_i(t)+0.5\delta\hat{\beta}_i(t)+0.5\psi_{ie}^2(t)+0.5\tilde{\bar{\chi}}_i^T(t)\tilde{\bar{\chi}}_i(t)+0.5\tilde{r}_i^2(t)+0.5\tilde{b}_i^T(t)\Gamma_{b_i}^{-1}\tilde{b}_i(t)+0.5(\eta(t)-\eta_f)^T(\eta(t)-\eta_f))\\ &\le \sum_{i=1}^{N}(\hat{\gamma}_i(t_0)+0.5\delta\hat{\beta}_i(t_0)+0.5\psi_{ie}^2(t_0)+0.5\tilde{\bar{\chi}}_i^T(t_0)\tilde{\bar{\chi}}_i(t_0)+0.5\tilde{r}_i^2(t_0)+0.5\tilde{b}_i^T(t_0)\Gamma_{b_i}^{-1}\tilde{b}_i(t_0)+\\ &0.5(\eta(t_0)-\eta_f)^T(\eta(t_0)-\eta_f))+\lambda_{\max}(\Gamma)\varepsilon_{\max}\|\eta(t_0)-\eta_f\|/\lambda_{\min}.\end{aligned} \tag{39}$$

The right hand side of (39) is bounded. Therefore the left hand side of (39) must also be bounded. This implies that the third inequality of (10) holds. Since $\lim_{t\to\infty}(\eta(t)-\eta_f)=0$, applying Barbalat's lemma to (30) gives

$$\lim_{t\to\infty}\frac{1}{1+\hat{\omega}(t)}\sum_{i=1}^{N}(\hat{\Omega}_i^T(t)\hat{C}\hat{\Omega}_i(t)+\rho_i\psi_{ie}^2(t)+0.5\tilde{\bar{\chi}}_i^T(t)\tilde{\bar{\chi}}_i(t)+0.5\tilde{r}_i^2(t))=0 \tag{40}$$

which implies that

$$\begin{cases}\lim_{t\to\infty}\hat{\Omega}_i^T(t)\hat{C}\hat{\Omega}_i(t)+\rho_i\psi_{ie}^2(t)+0.5\tilde{\bar{\chi}}_i^T(t)\tilde{\bar{\chi}}_i(t)+0.5\tilde{r}_i^2(t)=0\\ \lim_{t\to\infty}\hat{\omega}(t)=\sigma_1\end{cases} \tag{41}$$

or

$$\begin{cases} \lim_{t\to\infty} \hat{\Omega}_i^T(t)\hat{C}\hat{\Omega}_i(t) + \rho_i\psi_{ie}^2(t) + 0.5\tilde{\bar{\chi}}_i^T(t)\tilde{\bar{\chi}}_i(t) + 0.5\tilde{r}_i^2(t) = \sigma_2 \\ \lim_{t\to\infty} \hat{\omega}(t) = \infty \end{cases} \quad (42)$$

where σ_1 and σ_2 are some constants. From definitions of Ω_i and $\hat{\omega}$, the limit set (42) cannot be true. Therefore, the limit set (41) implies that $\lim_{t\to\infty} \|(\hat{\Omega}_i(t), \psi_{ie}(t), \tilde{\bar{\chi}}_i(t), \tilde{r}_i(t))\| = 0$. Therefore, carrying out the same analysis as in Section 3 yields the critical point $\hat{q} = \hat{q}_d$ is the asymptotically stable, and other equilibrium points are unstable or saddle. Furthermore, we can let the moving frame $\hat{O}\hat{X}\hat{Y}$ move along the trajectory $\mathbf{\Gamma_{od}}(x_{od}(s), y_{od}(s))$ by letting q_{od} move, i.e. by giving $\dot{s}$ some desired value since $\dot{q}_{od} = [x_{od}'(s)\ y_{od}'(s)]^T \dot{s}$. Finally, we note that convergence of $\hat{q}$ to $\hat{q}_d$ implies that of q_i to $R(\theta)^{-1}\hat{q}_{id} + q_{od} \triangleq q_{id}$, i.e. what we wanted to achieve. We summarize the results of this subsection in the following theorem.

Theorem 1. Under the assumptions stated in the control objective (see (9)), the control τ_i and the update law $\hat{\Theta}_i$ for unknown parameters given in (25), and the update law $\dot{\eta}$ for formation parameter (20) each agent i solves the control objective, i.e. (9) is achieved.

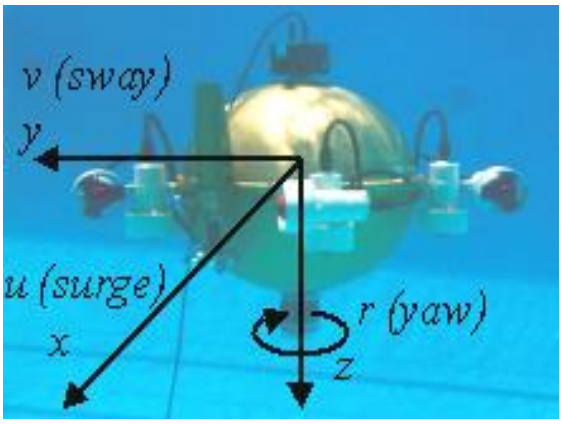

Figure 3. *An outside view of an ODIN. Courtesy http://www.eng.hawaii.edu/~asl/odinpics.*

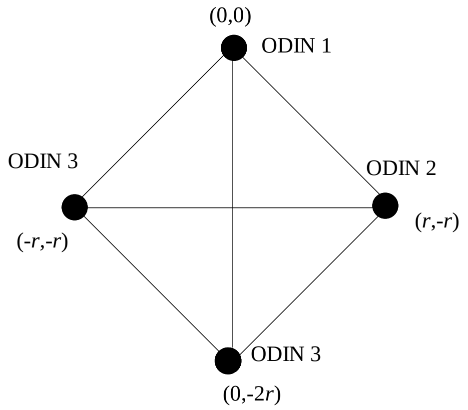

Figure 4. *Desired formation graph.*

SIMULATION RESULTS

We now illustrate the results stated in Theorem 2 by a simulation on formation tracking control for a fleet of 4 omni-directional intelligent navigators (ODINs) moving in a horizontal plane, see Figure 7.

The parameters of each ODIN are taken as follows $m_{i11} = m_{i22} = 300,\ m_{i33} = 16,\ X_{iu} = Y_{iv} = 168,\ N_{ir} = 10$. All other parameters defined in (3) are equal to zero. The disturbance vector is taken as $b_i = 0.5[m_{i11}\ m_{i22}\ m_{i33}]^T$. The safe distance between any two ODINs is $d_{ij} = 0.8$, $(i, j) \in \{1,2,3,4\}$. The control gains and tuning constants are chose as $\hat{C} = \text{diag}(3,3),\ \delta = 0.1,\ k = 1, w_i = 1,\ \rho_i = 1,$ $\Gamma_{bi} = \text{diag}(1,1,1)$ and the trajectory parameter s is updated with $\dot{s} = e^{-0.1\sum_{i=1}^{4}\|\hat{q}_i - \hat{q}_{id}\|}$. For clarity, we do not include the formation change in simulations, i.e. the formation parameter η is not used. The desired formation shape is specified as $\hat{q}_{1d} = [0,\ 0]^T,\ \hat{q}_{2d} = [r, -r]^T,\ \hat{q}_{3d} = [0,\ -2r]^T,\ \hat{q}_{4d} = [-r, -r]^T$ with $r = 15$, see Figure 8. We carry out two simulations. For the first simulation, the initial conditions are chosen as $q_1(0) = (-0.7r, 0),\ q_2(0) = (0, -0.7r),$ $q_3(0) = (0.7r, 0),\ q_4(0) = (0, 0.7r)$, and the reference trajectory q_{od} is a circle given by $q_{od}=[r\sin(s)\ r\cos(s)]^T$. A snap shot of motion of ODINs in the (x,y) plane is plotted in Figure 5. For clarity, we only plot the controls $\tau_1=[\tau_{u1}\ \tau_{v1}\ \tau_{r1}]^T$, and the distances from ODIN 1 to all other ODINs $\|q_1-q_i\|, i=2,3,4$ in Figure 6. For the second simulation, the initial conditions are chosen as $q_1(0)=(0,-1.7r),\ q_2(0)=(-0.7r,-r),\ q_3(0)=(0,-0.3r),\ q_4(0)=(0.7r,-r)$, and the reference trajectory q_{od} is a straight line given by $q_{od}=[s\ 0]^T$. A snap shot of motion of ODINs in the (x,y) plane is plotted in Figure 7. The controls $\tau_1=[\tau_{u1}\ \tau_{v1}\ \tau_{r1}]^T$, and the distances from ODIN 1 to all other ODINs $\|q_1-q_i\|, i=2,3,4$ are plotted in Figure 8. It is seen from these figures that all ODINs nicely form the desired formation and the desired formation graph moves on the desired reference trajectory. It is also seen that these distances are greater than $2d_{ij}, (i,j)\in\{1,2,3,4\}$, i.e. no collisions between the ODINs occur.

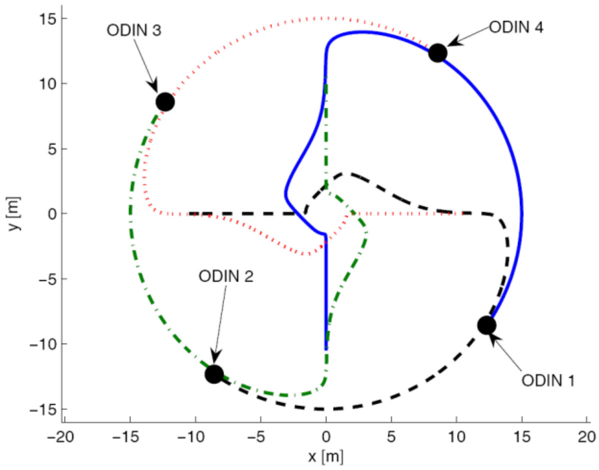

Figure 5. *Circular reference trajectory: a snap shot of motion of ODINs in (x,y) plane.*

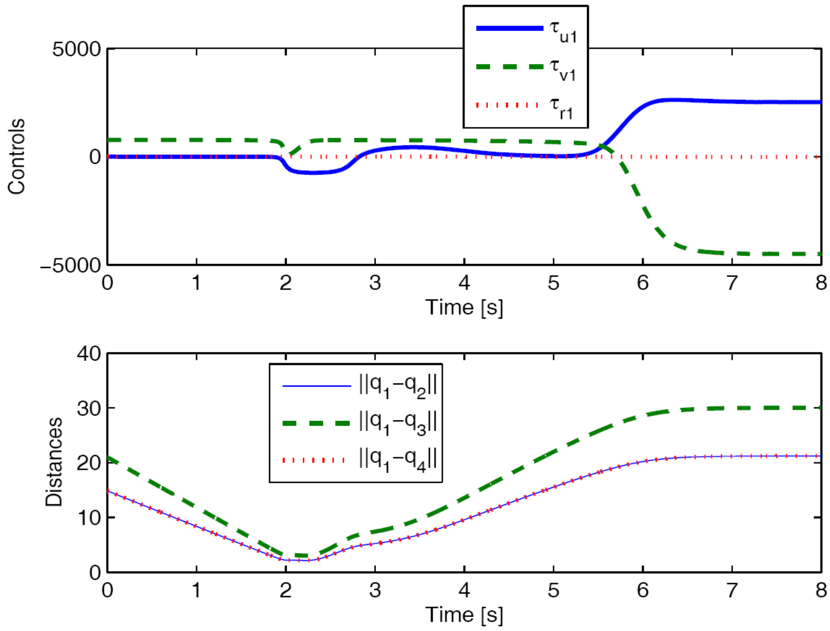

Figure 6. *Circular reference trajectory: Controls and distances from ODIN 1 to other ODINs*

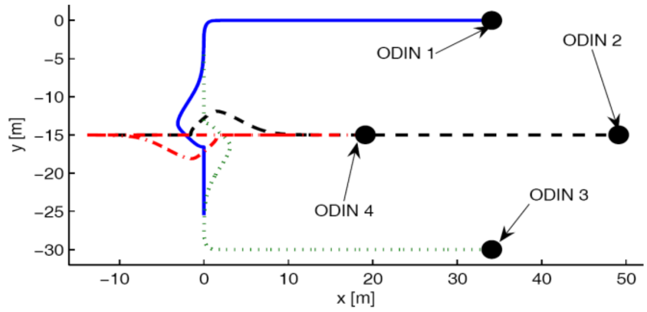

Figure 7. *Linear reference trajectory: a snap shot of motion of ODINs in (x,y) plane.*

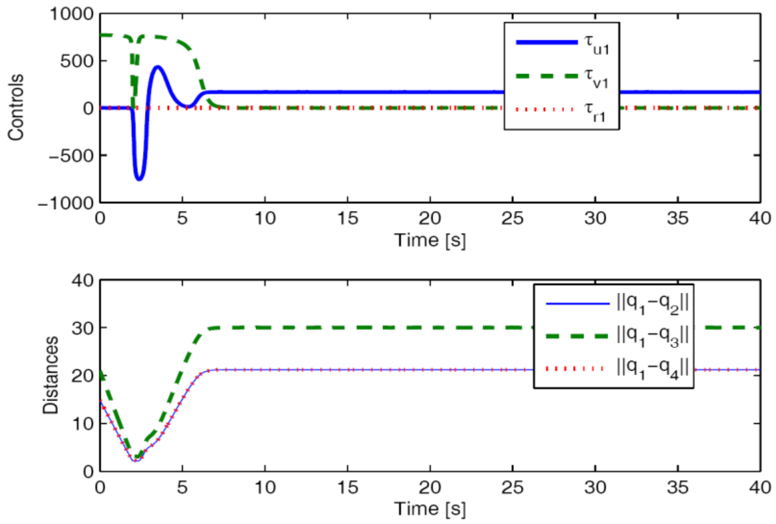

Figure 8. *Linear reference trajectory: Controls and distances from ODIN 1 to other ODINs*

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