A COMPARATIVE STUDY OF PTDF BASED-METHODS TO DETERMINE TRANSMISSION USAGE ALLOCATION FOR BILATERAL TRANSACTIONS IN POWER MARKETS

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ABSTRACT

The electricity industry has been deregulated with the aim of producing competition among power market participants, which would result in considerable difficulties for power system operators. One of these challenges is that power system operators need to give participants nondiscriminatory access to transmission usage. In this study, the transacted power flow in transmission networks for bilateral transactions deploying methods based Power Transfer Distribution Factor (PTDF) was analyzed and compared. The Power Transfer Distribution Factor can be classified as DCPTDF and ACPTDF. The results from these approaches using a three bus and Wood & Wollenberg six bus systems were elaborately computed and compared. The comparison shows that leveraging factors ACPTDF to allocate transmission usage is more exact but complicated than the techniques based factors DCPTDF. The comparative study in this paper provides a comprehensive evaluation that can support power system operators to achieve nondiscrimination in power trade.

SO SÁNH CÁC PHƯƠNG PHÁP SỬ DỤNG HỆ SỐ PTDF ĐỂ PHÂN BỔ CÔNG SUẤT TRÊN LƯỚI TRUYỀN TẢI CỦA CÁC GIAO DỊCH SONG PHƯƠNG TRONG THỊ TRƯỜNG ĐIỆN

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TỪ KHÓA

Thị trường điện

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ACPTDF

Phân bổ sử dụng truyền tải

Giao dịch song phương

TÓM TẮT

Ngành điện đã và đang tái cấu trúc với mục đích tạo ra sự cạnh tranh giữa những người tham gia thị trường điện. Sự tái cấu trúc này dẫn đến những khó khăn đáng kể cho các đơn vị vận hành hệ thống điện. Một trong những khó khăn này là các đơn vị vận hành hệ thống điện cần cung cấp cho đơn vị tham gia thị trường quyền sử dụng lưới điện truyền tải một cách công bằng. Trong nghiên cứu này, phân bố công suất trên lưới điện truyền tải của các giao dịch song phương được phân tích và so sánh sử dụng hệ số phân bố truyền tải công suất (PTDF). Hệ số phân bố truyền tải công suất được phân loại thành DCPTDF và ACPTDF. Kết quả tính toán từ cả hai tiếp cận này sử dụng các hệ thống điện 3 nút và 6 nút Wood & Wollenberg được phân tích và so sánh. Sự so sánh cho thấy phân bổ công suất trên lưới truyền tải của các giao dịch song phương khi áp dụng hệ số ACPTDF chính xác hơn nhưng phức tạp hơn so với sử dụng hệ số DCPTDF. Kết quả nghiên cứu trong bài báo này cung cấp sự đánh giá chi tiết để hỗ trợ các đơn vị vận hành hệ thống điện đạt được sự công bằng trong hoạt động thị trường điện.

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1. Introduction

Power deregulation aims at creating competitive electricity markets. In these markets, consumers have options to purchase electricity from various producers. Therefore, efficient transmission usage allocation needs to be studied to implement power transfer between diverse consumers and producers. Determining the extent of transmission service use aims to (1) equitably allocate transmission costs to those participating in the power grids and (2) calculate loss indicators to operate transmission grids more reliably and efficiently. Furthermore, fair transmission service is also essential in giving signals to the marketplace for long-term investment. Applying power flow methods such as Newton-Raphson, Fast Decoupled Power Flow (FDPF) to accurately compute the transmission usage level of market participants is complicated and time-consuming [1].

There are some effective approaches developed to deal with the problem of transmission usage allocation. In [2], the author proposed a topology-based approach to determine the share of a particular generator or demand in the power flow of each line branch. The paper [3] developed another technique based on concepts such as the domain of a generator, commons, links and state graph. The procedure using graph theory was put forward in [4] to determine injection factors of individual generators to line branch flows and withdrawal factors of individual demands from line flows. Authors in [5] introduced an AC power transfer distribution factor (PTDF)-based method to allocate the real power flow on transmission lines.

PTDFs are coefficients of the linear relationship between the power amount of contracts and power flow on lines [6]. PTDFs can be classified into two types, including DCPTDFs and ACPTDFs [7], in which DCPTDFs are determined using DC power flow, and ACPTDFs are the most comprehensive approximation of non-linear equations of power flow. In addition, a non-approximate dual methodology that uses power flows and topological cycles of networks was presented in [8] to compute PTDFs.

There are many applications pertaining to PTDFs. In [9] and [10], PTDFs based on AC Load Flow for multi-area Available Transfer Capability (ATC) determination and Linear Programming based on Optimal power flow analysis for locational marginal cost calculation was presented. Authors in [11] employed the PTDF index to suggest possible locations of new transmission lines that are likely to stabilize power systems. The congestion element of locational marginal prices can be determined using PTDF [12].

This paper aims to compare two techniques based on PTDF to allocate transmission service usage of bilateral contracts. This research has made major contributions as follow:

- Rigorously present a step-by-step procedure to calculate ACPTDF using a three-bus system;
- Compare the findings of transmission usage allocation from both kinds of sensitivity factors PTDF.

The paper is structured into four sections. Section 2 presents the Newton-Raphson method, formulations of DCPTDF and ACPTDF. Numerical results and discussions using three-bus and Wood & Wollenberg six-bus systems are given in Section 3, and the conclusions are inferred in Section 4.

2. Methodology

2.1. Newton-Raphson method

This section addresses the formulation and Newton-Raphson solution to the power flow problem. The complex current injected to the bus *i* is expressed as follow:

$$\dot{I}_i = \sum_{k=1}^{n} \dot{Y}_{ik} \dot{U}_k \qquad i = 1, 2, ..., N \tag{1}$$

where $\dot{U}_k$ is the complex voltage at node *k*; $\dot{Y}_{ik} = G_{ik} + jB_{ik}$ is the *ik*th element of the admittance matrix, and *N* is the total nodes of the power system with N_G generator buses and N_D load nodes. Bus 1 is chosen as the voltage reference bus.

The complex power at *i*th bus:

$$\dot{S}_i = \dot{U}_i \left(\sum_{k=1}^{n} \dot{Y}_{ik} \dot{U}_k \right)^* = \dot{U}_i \sum_{k=1}^{n} Y_{ik}^* U_k^* \quad \forall i \tag{2}$$

Using rectangular coordinates for elements of the admittance matrix and polar form for voltages leads to:

$$\dot{S}_i = U_i \sum_{k=1}^{n} U_k (G_{ik} - jB_{ik}) \left(\cos\delta_{ik} + j \sin\delta_{ik} \right) \ \forall i \tag{3}$$

where δ_{ik} is the difference between the phase angles of nodes *i* and *k*; U_i, U_k is the voltage magnitude at bus *i* and *k*, respectively.

Split (3) into 2*n* real equations:

$$\begin{cases} P_i = U_i \sum_{k=1}^{n} U_k (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) \\ Q_i = U_i \sum_{k=1}^{n} U_k (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}) \end{cases} \tag{4}$$

The Newton-Raphson (NR) successively improves unknown values through the first-order approximation of the involved non-linear equations. According to NR method, with initial values, corrections are obtained by solving the linear equation system:

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix}^{(r)} = \mathbf{J}^{(r)} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta \mathbf{U} \end{bmatrix}^{(r)} \tag{5}$$

where **J** is the Jacobian matrix; $\Delta\mathbf{P}$ and $\Delta\mathbf{Q}$ are the difference between the specified power and the power computed with the most recent values.

The iteration continues until all the mismatch vector of power in absolute value lower than the pre-specified tolerance.

2.2. DC Power Flow

This DC power flow (DCPF) method is developed with assumptions as follows:

- the voltage magnitude at all buses is equal to 1;
- the phase angle difference corresponding to adjacent buses are small;
- series resistance is ignored.

With these hypotheses, the real power flow is simplified to

$$P_{ik} = \frac{1}{x_{ik}} \left(\delta_i - \delta_k \right) \tag{6}$$

where x_{ik} is the reactance of *ik*th branch.

The linear relationship between nodal power injection and voltage angle, according to the DCPF method, is written as follows:

$$P_i = \sum_k P_{ik} = \sum_k \frac{1}{x_{ik}} \left(\delta_i - \delta_k \right) \tag{7}$$

2.3. DCPTDF formulation

Equation (7) can be rewritten in matrix form as $\mathbf{P} = \mathbf{B}\boldsymbol{\delta}$. The relationship between branch power flow $\mathbf{P}_f$ and nodal injected power **P** can be obtained by eliminating phase angles:

$$\begin{aligned}\mathbf{A}^T\boldsymbol{\delta} = \mathbf{X}\mathbf{P}_f \Rightarrow \mathbf{P}_f = \left[\mathbf{X}^{-1}\mathbf{A}^T\right]\boldsymbol{\delta} \\ \mathbf{P} = \mathbf{A}\mathbf{P}_f \Rightarrow \mathbf{P}_f = \left[\mathbf{X}^{-1}\mathbf{A}^T\mathbf{B}^{-1}\right]\mathbf{P} = \mathbf{S}_f\mathbf{P}\end{aligned} \quad (8)$$

where **A** is the reduced branch-to-node incidence matrix omitting the swing node, **X** is a diagonal matrix with elements of branch reactances, $\mathbf{S}_f$ is the matrix of sensitivities between branch power flows and nodal powers.

Then, the power flow on the branches after a change in nodal power can be computed as follows:

$$\mathbf{P}_f = \mathbf{P}_f^0 + \mathbf{S}_f\left(\mathbf{P} - \mathbf{P}^0\right) \Rightarrow \Delta\mathbf{P}_f = \mathbf{S}_f\Delta\mathbf{P} \quad (9)$$

Power transfer distribution factor (PTDF) is defined as a power flow increase on the *mn* line when the power injected in *i*th node increases by 1 MW

$$PTDF_{mn-i} = \frac{\Delta P_{mn}^i}{\Delta P_i} = \frac{\Delta\delta_m - \Delta\delta_n}{x_{mn}} = S_{mn-i} \quad (10)$$

The PTDF above depends solely on the structure of electrical networks; therefore, these factors can be calculated off-line using sparse matrix techniques. However, the PTDF depends on the location of the voltage reference node.

2.4. ACPTDF formulation

The sensitivity factors ACPTDF are applied to determine a change in the power flow of branches after changing in power transactions at different operating states from the sensitivity values of the Jacobian matrix in the Newton-Raphson algorithm.

Consider a bilateral transaction P_t between the seller at bus *i* and the buyer at bus *j*. Furthermore, consider a line *mn* which is connected between nodes *m* and *n* and transfers the part of the transacted power. When the active power contract ($\Delta P_{t-i,j}$) between the above market participants is changed, the variation in the power amount of transmission line *mn* (ΔP_{mn}) is calculated as

$$\Delta P_{mn} = \text{ACPTDF}_{mn-i,j}\Delta P_{t-i,j} \quad (11)$$

Equation (5) can be expanded as (12). The change in power flow of line *mn* can be determined using sensitivity analysis as (13).

$$\left[\Delta\delta_2 \quad \cdots \quad \Delta\delta_N \quad \Delta U_{N_G+1} \quad \cdots \quad \Delta U_{N_G+N_D}\right]^T = (\mathbf{J})^{-1}\left[\Delta P_2 \quad \cdots \quad \Delta P_N \quad \Delta Q_{N_G+1} \quad \cdots \quad \Delta Q_{N_G+N_D}\right]^T \quad (12)$$

$$\Delta P_{mn} = \left(\frac{\partial P_{mn}}{\partial\delta_2}\ldots\frac{\partial P_{mn}}{\partial\delta_n}\frac{\partial P_{mn}}{\partial U_{N_G+1}}\ldots\frac{\partial P_{mn}}{\partial U_{N_G+N_D}}\right)\left[\Delta\delta_2 \quad \cdots \quad \Delta\delta_N \quad \Delta U_{N_G+1} \quad \cdots \quad \Delta U_{N_G+N_D}\right]^T \quad (13)$$

Substituting equation (12) in equation (13), the change in power flow of line *mn* can be calculated as equation (14).

$$\Delta P_{mn} = \left(\frac{\partial P_{mn}}{\partial\delta_2}\ldots\frac{\partial P_{mn}}{\partial\delta_N}\frac{\partial P_{mn}}{\partial U_{N_G+1}}\ldots\frac{\partial P_{mn}}{\partial U_{N_G+N_D}}\right)(\mathbf{J})^{-1}\left[\Delta P_2 \quad \cdots \quad \Delta P_N \quad \Delta Q_{N_G+1} \quad \cdots \quad \Delta Q_{N_G+N_D}\right]^T \quad (14)$$

A bilateral contract is defined by a tuple (t, i, j, P_t), in which t is the contract number, i and j are the seller and buyer buses, respectively, and P_t is the transacted power. For a bilateral transaction t:

$$\Delta P_i = +P_t;\ \Delta P_j = -P_t;\ \Delta P_k = 0;\ \Delta Q_k = 0 \quad k = 2,\ldots,N;\ k \neq i, j \quad (15)$$

The linear factors ACPTDF can be obtained from the following equation:

$$\text{ACPTDF}_{mn-i,j} = \left(\frac{\partial P_{mn}}{\partial\delta_2}\ldots\frac{\partial P_{mn}}{\partial\delta_N}\frac{\partial P_{mn}}{\partial U_{N_G+1}}\ldots\frac{\partial P_{mn}}{\partial U_{N_G+N_D}}\right)(\mathbf{J})^{-1} \quad (16)$$

Incorporating (15) into (14), the change in branch power flow due to bilateral transactions can be determined. The ACPTDF are determined at a base power flow condition and are deployed to compute the change in power flow of transmission lines at other operational conditions.

3. Results and discussion

In this section, the transmission usage allocation using ACPTDF and ACPTDF methods is calculated on three-bus and Wood & Wollenberg six bus systems.

3.1. Three-bus system

The diagram of a three-bus system is depicted in Figure 1. The first bus is considered the slack bus. The bus data and line data are shown in Table 1 and Table 2, respectively.

Table 1. *Bus data for three-bus system*

Bus	U	P_D	Q_D	P_G	Q_G
1	1.05	0	0	-	-
2	1.05	6	3	3.8	-
3	1.07	2	1.2	1.7	-

Table 2. *Line data for three-bus system*

Line	r	x	$b/2$
1-2	0.02	0.1	0.01
1-3	0.02	0.1	0.01
2-3	0.01	0.05	0.005

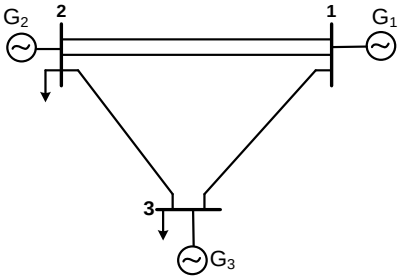

Figure 1. *Three-bus system*

3.1.1 Sensitivity factors DCPTDF

Matrices **A**, **X** and **B** are as follows

$$\mathbf{X}=\begin{bmatrix}0.05 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.05\end{bmatrix} \qquad \mathbf{A}=\begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & -1\end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}40 & -20\\ -20 & 30\end{bmatrix} \qquad \mathbf{DCPTDF}=\mathbf{S}_f=[\mathbf{X}]^{-1}\mathbf{A}^T[\mathbf{B}]^{-1}=\begin{bmatrix}-0.75 & -0.5\\ -0.25 & -0.5\\ 0.25 & -0.5\end{bmatrix}$$

3.1.2 Sensitivity factors ACPTDF

The final results of bus voltages are $\dot{U}_1=1,05;\dot{U}_2=1,05\angle-0,0771;\dot{U}_3=1,07\angle-0,0399$

Jacobian matrix:

$$\mathbf{J}=\begin{bmatrix}\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial U_2} & \frac{\partial P_2}{\partial U_3}\\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial U_2} & \frac{\partial P_3}{\partial U_3}\\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial U_2} & \frac{\partial Q_2}{\partial U_3}\\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial U_2} & \frac{\partial Q_3}{\partial U_3}\end{bmatrix}=\begin{bmatrix}42.24 & -21.43 & 5.7 & -4.79\\ -21.75 & 32.46 & -3.35 & 6.64\\ -10.99 & 5.13 & 40.49 & -20.03\\ 3.52 & -6.11 & -20.71 & 31.36\end{bmatrix}$$

The sensitivity matrix of line power flow with respect to state variables can be expressed as follows:

$$\mathbf{D}=\begin{bmatrix} \frac{\partial P_{12}}{\partial \delta_2} & \frac{\partial P_{12}}{\partial \delta_3} & \frac{\partial P_{12}}{\partial U_2} & \frac{\partial P_{12}}{\partial U_3} \\ \frac{\partial P_{13}}{\partial \delta_2} & \frac{\partial P_{13}}{\partial \delta_3} & \frac{\partial P_{13}}{\partial U_2} & \frac{\partial P_{13}}{\partial U_3} \\ \frac{\partial P_{23}}{\partial \delta_2} & \frac{\partial P_{23}}{\partial \delta_3} & \frac{\partial P_{23}}{\partial U_2} & \frac{\partial P_{23}}{\partial U_3} \end{bmatrix}=\begin{bmatrix} -21.465 & 0 & 3.711 & 0 \\ 0 & -10.886 & 0 & -1.611 \\ 21.429 & -21.429 & 3.204 & -4.790 \end{bmatrix}$$

The sensitivity matrix of factors ACPTDF (only consider active power):

$$\mathbf{ACPTDF}=\mathbf{D}.\mathbf{J}^{-1}=\begin{bmatrix} -0.708 & -0.477 \\ -0.26 & -0.504 \\ 0.253 & -0.495 \end{bmatrix}$$

The differences between the sensitivity factors DCPTDF and ACPTDF are determined according to (17) and are shown in Table 3.

$$D(\%)=\frac{\text{DCPTDF}-\text{ACPTDF}}{\text{ACPTDF}}.100 \quad (17)$$

Table 3. *Difference (in percentage) between DCPTDF and ACPTDF for 3-bus system*

ACPTDF		DCPTDF		$D(\%)$	
-0.708	-0.477	-0.75	-0.5	-2.583	-1.628
-0.26	-0.504	-0.25	-0.5	-4.438	-1.588
0.253	-0.495	0.25	-0.5	-1.419	1.471

The obtained results from a three-bus system above show that the sensitivity factors PTDF with the DC model are very close to that of the AC model (the difference is less than 5%). On the other hand, in terms of computational performance, computing ACPTDF is more complex when compared to DCPTDF.

3.2. Wood & Wollenberg six-bus system

This section analyzes the results obtained with a Wood & Wollenberg six-bus system [13]. The first bus is the swing bus.

The obtained results for linear factors DCPTDF and ACPTDF are illustrated in Table 4 and Table 5, respectively. In these tables, the numbers such as (1) in the first row and (1-2) in the first column represent the bus and transmission line, respectively.

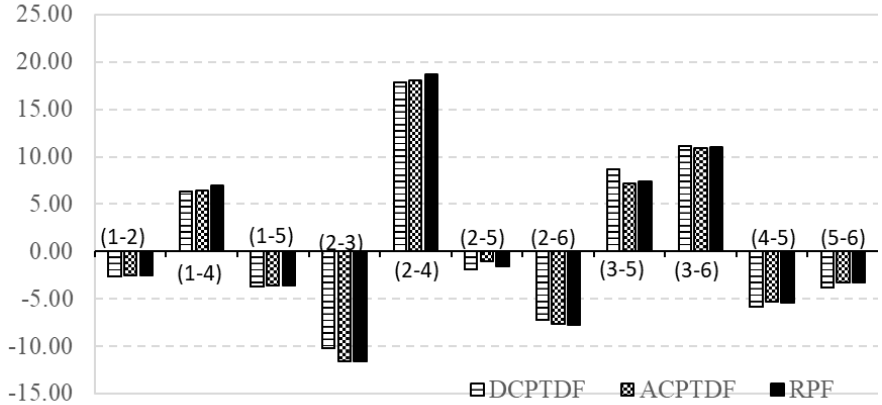

Figure 2. *The change of line power flow (in* MW*) for bilateral transaction*

Table 4. *Linear factors DCPTDF*

	2	3	4	5	6
1-2	-0.4706	-0.4026	-0.3149	-0.3217	-0.4064
1-4	-0.3149	-0.2949	-0.5044	-0.2711	-0.296
1-5	-0.2145	-0.3026	-0.1807	-0.4072	-0.2976
2-3	0.0544	-0.3416	0	-0.1057	-0.1907
2-4	0.3115	0.2154	-0.379	0.1013	0.2208
2-5	0.0993	-0.0342	0.0292	-0.1927	-0.0266
2-6	0.0642	-0.2422	0	-0.1246	-0.41
3-5	0.0622	0.289	0	-0.1207	0.1526
3-6	0	0.3695	0	0	-0.3433
4-5	0	-0.0795	0.1166	-0.1698	-0.0752
5-6	-0.0565	-0.1273	0	0.1096	-0.2467

Table 5. *Linear factors ACPTDF*

	2	3	4	5	6
1-2	-0.4457	-0.3887	-0.3054	-0.3198	-0.4003
1-4	-0.3235	-0.3048	-0.5202	-0.286	-0.3129
1-5	-0.2214	-0.3056	-0.1873	-0.4174	-0.308
2-3	0.0618	-0.3674	0.02	-0.1185	-0.2144
2-4	0.3151	0.2359	-0.367	0.1235	0.243
2-5	0.1044	0	0.0351	-0.1899	0
2-6	0.0643	-0.2326	0.0207	-0.1241	-0.402
3-5	0.0654	0.2616	0.0213	-0.1239	0.1468
3-6	0	0.363	0	0	-0.3541
4-5	0	-0.0676	0.1108	-0.1597	-0.0686
5-6	-0.0594	-0.1304	-0.0198	0.1092	-0.244

A bilateral contract (1, 3, 4, 30 MW) is implemented between the power plant at node 3 (source node) and the load at node 4 (sink node) with a transacted power of 30 MW. The purpose of implementing this scenario is to calculate and compare the power flow change (values and directions) using three approaches, including DCPTDF, ACPTDF, and Repeated Power Flow (RPF). The results of power flow change in each line are illustrated in Figure 2. The negative values indicate that the actual power flow is in the reserve direction.

As shown in Figure 2, this bilateral transaction can significantly impact the power flow change of lines 2-3, 2-4 and 3-6, while the change of power flow in lines 2-5 and 1-2 is considerably low. Furthermore, the obtained outcomes from the ACPTDF-based method are closer to the RPF method than that of the DCPTDF-based technique.

4. Conclusion

This paper studies both methodologies, namely DCPTDF and ACPTDF, to calculate transmission usage of bilateral contracts. The results show that the solution obtained with the ACPTDF is more accurate than that of DCPTDF and is very close to the solution obtained with the repeated power flow method. These findings provide valuable information for Independent System Operators (ISO) and Market Operators (MO) to allocate transmission usage service equitably, which plays a vital role in operating electricity markets effectively.

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