AN ALGORITHM TO APPROXIMATE DOUBLE INTEGRALS BY USING ADAPTIVE QUADRATURE METHOD

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ABSTRACT

For the numerical integration in one variable, the Adaptive quadrature is a well-known method, which is superior in the sense of reducing the number of the evaluations of the function than other methods using equally-space nodes, and thus increases the accuracy of the approximation. This method takes the functional variation into account when controlling the subdivision of the given interval into suitable step sizes, in which the subintervals with larger variation have smaller step sizes in regarding that the obtained approximation is within a given specified tolerance. That is, this procedure distributes the error uniformly into equal subintervals. In this article, we develop an algorithm applying Adaptive quadrature method, which bases on the Composite Simpson's rule, to approximate double integrals over a general region in the plane. We will prove that this algorithm works for double integrals. Besides, the article gives the pseudocode for the algorithm and an interesting example to illustrate the use of the algorithm. The examples is implemented by using Matlab code.

Keywords: *double integrals, numerical integration, Adaptive quadrature method, Composite Simpson's rule, quadrature.*

INTRODUCTION

The methods of calculating the numerical double integrals are developed by basing on ones of the numerical integrals in one variable. They include the numerical quadrature methods such as Trapezoidal Rule, Simpson's Rule (or more general, Newton-Cotes formulas), Composite Methods, Romberg's Method, and Adaptive quadrature. The quadrature methods use the sum of the form

$$\sum_{i=1}^{n} a_i f(x_i)$$

to approximate the integral $\int_a^b f(x)\,dx$, where the coefficients a_i and the nodes $x_i \in [a,b]$ are chosen in an appropriate way. Adaptive quadrature is one of quadrature methods, which is often a good choice in considering to reduce the evaluation of function at mesh points, so it decreases the round-off errors. In this article, we are going to develop the latter to the numerical double integrals. First, we fix a tolerance $\varepsilon > 0$, and check that whether the accuracy of the error estimate is within the given tolerance ε. If it is, then we can take that approximation of the integral. If it is not, we divide the region of integration into four subregions, then determine if the error on each subregion is within the tolerance $\varepsilon/4$. If the error on a subregion is not, then we continue dividing it into four smaller subregions and require the tolerance on each now is within $\varepsilon/4^2$. That means, we distribute the error basing on the variation of the function integrated on each small region. The process is continued until we reach the requirement.

DOUBLE INTEGRALS OVER RECTANGLES

First of all, we consider the simple case, where the region of integration is only a rectangle. Assume that we want to approximate the double integral

$$I = \iint\limits_{[a,b]\times[c,d]} f(x,y)dA = \int\limits_a^b \int\limits_c^d f(x,y)dydx.$$

For each $x \in [a,b]$, by Simson's Rule for the integral of one variable, for some $\xi_x \in (c,d)$,

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$$\int_c^d f(x,y)dy = \frac{k}{3}[f(x,y_0) + 4f(x,y_1) + f(x,y_2)] - \frac{k^5}{90}\frac{\partial^4 f}{\partial y^4}(x,\xi_x),$$

$\int_a^b f(x,y_i)dx = \frac{h}{3}[f(x_0,y_i) + 4f(x_1,y_i) + f(x_2,y_i)] - \frac{h^5}{90}\frac{\partial^4 f}{\partial x^4}(\eta_i,\xi_{x_i}), \forall i = 0,1,2.$

where $h = \frac{b-a}{2}, k = \frac{d-c}{2}, y_0 = c, y_1 = c + k, y_2 = c + 2k = d, x_0 = a, x_1 = a + h, x_2 = a + 2h = b.$

So, $I = \frac{hk}{9}\{[f(x_0,y_0) + 4f(x_1,y_0) + f(x_2,y_0)] + 4[f(x_0,y_1) + 4f(x_1,y_1) + f(x_2,y_1)] + [f(x_0,y_2) + 4f(x_1,y_2) + f(x_2,y_2)]\} - E_1 =: S_1 - E_1.$ (1)

where the error estimate is

$E_1 := \frac{k}{3}\frac{h^5}{90}\left[\frac{\partial^4 f}{\partial x^4}(\eta_0,\xi_{x_0}) + 4\frac{\partial^4 f}{\partial x^4}(\eta_1,\xi_{x_1}) + \frac{\partial^4 f}{\partial x^4}(\eta_2,\xi_{x_2})\right] + \frac{k^5}{90}\int_a^b \frac{\partial^4 f}{\partial y^4}(x,\xi_x)dx.$

By Mean Value Theorem, there exists $\bar{\mu} \in [a,b]$ such that

$$\int_a^b \frac{\partial^4 f}{\partial y^4}(x,\xi_x)dx = (b-a)\frac{\partial^4 f}{\partial y^4}(\bar{\mu},\xi_{\bar{\mu}}).$$

By the Intermediate Value Theorem, there exists $(\bar{\eta},\bar{\xi}) \in [a,b]\times[c,d]$ such that

$\frac{\partial^4 f}{\partial x^4}(\eta_0,\xi_{x_0}) + 4\frac{\partial^4 f}{\partial x^4}(\eta_1,\xi_{x_1}) + \frac{\partial^4 f}{\partial x^4}(\eta_2,\xi_{x_2}) = 6\frac{\partial^4 f}{\partial x^4}(\bar{\eta},\bar{\xi}).$

So, $E_1 = \frac{2h^5 k}{90}\frac{\partial^4 f}{\partial x^4}(\bar{\eta},\bar{\xi}) + \frac{k^5}{90}(b-a) \times \frac{\partial^4 f}{\partial y^4}(\bar{\mu},\xi_{\bar{\mu}}) = \frac{(b-a)(d-c)}{180}\left[h^4\frac{\partial^4 f}{\partial x^4}(\bar{\eta},\bar{\xi}) + k^4\frac{\partial^4 f}{\partial y^4}(\bar{\mu},\xi_{\bar{\mu}})\right].$

By Composite Simpson's Rule, with $p = \frac{b-a}{4}, q = \frac{d-c}{4}, x_0 = a, y_0 = c, x_i = x_0 + ip, y_i = y_0 + iq, i = 0,1,\ldots,4,$ for each $x \in [a,b]$, there is $\gamma_x \in (c,d)$ such that

$\int_c^d f(x,y)dy = \frac{q}{3}[f(x,y_0) + 4f(x,y_1) + 2f(x,y_2) + 4f(x,y_3) + f(x,y_4)] - \frac{(d-c)q^4}{180}\frac{\partial^4 f}{\partial y^4}(x,\gamma_x),$

$\int_a^b f(x,y_i)dy = \frac{p}{3}[f(x_0,y_i) + 4f(x_1,y_i) + 2f(x_2,y_i) + 4f(x_3,y_i) + f(x_4,y_i)] - \frac{(b-a)p^4}{180}\frac{\partial^4 f}{\partial x^4}(\beta_i,\gamma_{x_i}), \forall i = 0,1,\ldots,4.$

So, $I = \frac{pq}{9}\sum_{i=0}^4 m_i[f(x_0,y_i) + 4f(x_1,y_i) + 2f(x_2,y_i) + 4f(x_3,y_i) + f(x_4,y_i)] - E_2$

$=: S_2 - E_2.$ (2)

where $m_0 = m_4 = 1, m_1 = m_3 = 4, m_2 = 2,$ and the error estimate

$E_2 := \frac{q}{3}\frac{(b-a)p^4}{180}\sum_{i=0}^4 m_i \frac{\partial^4 f}{\partial x^4}(\beta_i,\gamma_{x_i}) + \frac{(d-c)q^4}{180}\int_a^b \frac{\partial^4 f}{\partial y^4}(x,\gamma_x)dx.$

Similar to E_1, by the Mean Value Theorem and the Intermediate Value Theorem, we can rewrite

$$E_2 = \frac{(b-a)(d-c)}{180}\left[p^4\frac{\partial^4 f}{\partial x^4}(\hat{\beta},\hat{\gamma}) + q^4\frac{\partial^4 f}{\partial y^4}(\hat{\mu},\gamma_{\hat{\mu}})\right],$$

for some $(\hat{\beta},\hat{\gamma}), (\hat{\mu},\gamma_{\hat{\mu}}) \in [a,b]\times[c,d]$.

Compare (1) and (2) with $h = 2p, k = 2q$ and assume that $(\bar{\eta},\bar{\xi}) \approx (\hat{\beta},\hat{\gamma}), (\bar{\mu},\xi_{\bar{\mu}}) \approx (\hat{\mu},\gamma_{\hat{\mu}})$, we have $S_2 - I = E_2, S_1 - I = E_1, E_1 \approx 16E_2$. Therefore, $|S_2 - I| \approx \frac{1}{15}|S_1 - S_2|$. That is, S_2 approximates I about 15 times better than it agrees with S_1. So, if $|S_1 - S_2| < 15\varepsilon$, we can believe that $|I - S_2| = \left|\iint_{[a,b]\times[c,d]} f(x,y)dxdy - S_2\right| < \varepsilon$, and be confident enough to approximate the double integral I by S_2.

If $|S_1 - S_2| \geq 15\varepsilon$, we divide the rectangle into four subrectangles by mesh points

$$a = x_0 < x_1 = \frac{a+b}{2} < x_2 = b,$$

$$c = y_0 < y_1 = \frac{c+d}{2} < y_2 = d.$$

Now, on each subrectangle, we required that

$$\left|S_1^{subrectangle} - S_2^{subrectangle}\right| < \frac{15\varepsilon}{4} \quad (3)$$

and expect that this assures the requirement $\left|S_2^{subrectangle} - I^{subrectangle}\right| < \varepsilon/4$. Here, $S_1^{subrectangle}$ and $S_2^{subrectangle}$ are approximations for the value of the double integral over the considered subrectangle obtained by applying (1) and (2) on this subrectangle, respectively, and denote $I^{subrectangle}$ the exact value of this double integral. We continue the above process. That is, if the condition (3) is not met, we persist in subdiving this subrectangle into four smaller subrectangles. And if it is met, then accept confidently the approximation

$$S_2^{subrectangle} \approx I^{subrectangle}.$$

DOUBLE INTEGRALS OVER GENERAL REGIONS

We limit our consideration to the region of integration with the following form

$$\Omega = \{(x,y) | a \le x \le b, c(x) \le y \le d(x)\},$$

where $c(x), d(x)$ are functions defined on $[a,b]$. By setting $2p = h = \frac{b-a}{2}, 2q(x) = k(x) = \frac{d(x)-c(x)}{2}$ $(\forall x \in [a,b])$, and rewriting $\iint_\Omega f(x,y)dA = \int_a^b \int_{c(x)}^{d(x)} f(x,y)dydx$, we can modify (1) and (2) to get

$$I = \frac{h}{9}\sum_{i=0}^{2} n_i \left[\sum_{j=0}^{2} n_j k(x_j) f(x_j, y_i)\right] - \widehat{E_1}$$
$$=: \widehat{S_1} - \widehat{E_1},$$

$$I = \frac{p}{9}\sum_{i=0}^{4} m_i \left[\sum_{j=0}^{4} m_j q(x_j) f(x_j, y_i)\right] - \widehat{E_2}$$
$$=: \widehat{S_2} - \widehat{E_2}.$$

where $n_0 = n_2 = 1, n_1 = 4$, and $m_0 = m_4 = 1, m_2 = 2, m_3 = m_4 = 4$, the error estimates

$\widehat{E_1} = \frac{b-a}{90}\left[h^4 k(\bar{\eta}) f(\bar{\eta}, \bar{\xi}) + k^5(\bar{\mu}) \frac{\partial^4 f}{\partial y^4}(\bar{\mu}, \gamma_{\bar{\mu}})\right]$, and

$\widehat{E_2} = \frac{(b-a)}{45}\left[p^4 q(\hat{\eta}) \frac{\partial^4 f}{\partial x^4}(\hat{\eta}, \hat{\xi}) + q^4(\hat{\mu}) \frac{\partial^4 f}{\partial y^4}(\hat{\mu}, \gamma_{\hat{\mu}})\right] = \frac{1}{16}\frac{b-a}{90}\left[h^4 k(\hat{\eta}) \frac{\partial^4 f}{\partial x^4}(\hat{\eta}, \hat{\xi}) + k^4(\hat{\mu}) \frac{\partial^4 f}{\partial y^4}(\hat{\mu}, \gamma_{\hat{\mu}})\right]$,

for some $(\bar{\eta}, \bar{\xi}), (\bar{\mu}, \gamma_{\bar{\mu}}), (\hat{\eta}, \hat{\xi}), (\hat{\mu}, \gamma_{\hat{\mu}}) \in \Omega$. By assuming that $(\bar{\eta}, \bar{\xi}) \approx (\hat{\eta}, \hat{\xi}), (\bar{\mu}, \gamma_{\bar{\mu}}) \approx (\hat{\mu}, \gamma_{\hat{\mu}})$, we get $\widehat{E_2} \approx \frac{1}{16}\widehat{E_1}$, and thus $\left|\widehat{S_2} - I\right| \approx \frac{1}{15}\left|\widehat{S_2} - \widehat{S_1}\right|$. So the above process in the case of integrated rectangular region can be reapplied to get the requirement for the accuracy of the approximation.

Remark 1. A similar process can be use to approximate the double integrals where the integrated region can be described as $\widetilde{\Omega} = \{(x,y) | c \le y \le d, a(y) \le x \le b(y)\}$, where $a(y), b(y)$ are functions of y, defined on $[c,d]$.

Remark 2. The error estimate is vanished if the partial derivatives $\frac{\partial^4 f}{\partial x^4}$ and $\frac{\partial^4 f}{\partial y^4}$ are both vanished. In this case, Simpson's approximation become exact. Therefore, the Adaptive quadrature method stop by the first recursion and give an exact result for the integral. Illustration for such a function is a polynomial of two variable x, y of the degree with respect to only x and y less than 4. For example, $f(x,y) = x^2 + 3xy^3 + y^3$.

ALGORITHM

The following pseudocode describes the algorithm for the Adaptive quadrature procedure based on Simpson's Rule to approximate the double integral over a rectangle

$$I = \iint_{[a,b]\times[c,d]} f(x,y)dA.$$

One can easily construct the algorithm for the case where the integrated region is generally describes as in the above Section.

INPUT rectangle a, b, c, d, tolerance ϵ, limit N to number of levels.

OUTPUT approximation AP of I, or message that N is exceeded (the procedure fails).

Step 1 (Initiate the procedure)

$AP := 0; i := 0, L_i := 1; \epsilon_i := 15\epsilon$; (Here, L_i indicates the recent level of the subdivision.)

$R_{i1} := a; R_{i2} := b; R_{i3} := c; R_{i4} := d$; (Data for the initiated rectangle.)

$n_0 := 1; n_1 := 4; n_2 := 1$; (These are the coefficients in Simpson's Rule.)

$m_0 := 1; m_1 := 4; m_2 := 2; m_3 := 4; m_4 := 1$. (These are the coefficients in Composite Simpson's Rule.)

Step 2 While $i > 0$ do Steps 3-5.

Step 3

$h_i := 0.5(R_{i2} - R_{i1}); k_i := 0.5(R_{i4} - R_{i3})$;

For $j = 0$ to 2 do

$$x_j := R_{i1} + jh_i; y_j := R_{i3} + jk_i$$

(Set up data for the mesh points.)

End do;

$S_1 := 0; S_2 := 0$; (Initiate the values for Simpson's and Composite Simpson's Rule.)

For $l = 0$ to 2 do

For $j = 0$ to 2 do

$$S_1 := S_1 + \frac{h_i k_i}{9} n_l n_j f(x_l, x_j)$$

(Simpson's Rule)

End do;

End do;

For $l = 0$ to 4 do

For $j = 0$ to 4 do

$$S_2 := S_2 + \frac{h_i k_i}{36} m_l m_j f(x_l, x_j).$$

(Composite Simpson's Rule)

End do;

End do;

$$u_1 := R_{i1}; u_2 := R_{i2}; u_3 := R_{i3}; u_4 := R_{i4};$$

(Save data.)

$$u_5 := h_i; u_6 := k_i; u_7 := \epsilon_i; u_8 := L_i.$$

Step 4 $i := i - 1$. (Delete the level.)

Step 5 If $|S_2 - S_1| < u_7$ then

$$AP := AP + S_2$$

Else

if $u_8 \geq N$ then

OUTPUT("LEVEL IS EXCEEDED.");

STOP.

else (Add one level.)

$i := i + 1$;

(Data for the Subrectangle R_1, cf. Figure 1.)

$R_{i1} := u_1; R_{i2} := u_1 + 0.5u_5$;

$R_{i3} := u_3; R_{i4} := u_3 + 0.5u_6$;

$\epsilon_i := \frac{u_7}{4}; L_i := u_8 + 1$;

$i := i + 1$;

(Data for the Subrectangle R_2.)

$R_{i1} := u_1; R_{i2} := u_1 + 0.5u_5$;

$R_{i3} := u_3 + 0.5u_6; R_{i4} := u_4$;

$\epsilon_i := \epsilon_{i-1}; L_i := L_{i-1}$;

$i := i + 1$;

(Data for the Subrectangle R_3.)

$R_{i1} := u_1 + 0.5u_5; R_{i2} := u_2$;

$R_{i3} := u_3; R_{i4} := u_3 + 0.5u_6$;

$\epsilon_i := \epsilon_{i-1}; L_i := L_{i-1}$;

$i := i + 1$;

(Data for the Subrectangle R_4.)

$$R_{i1} := u_1 + 0.5u_5; R_{i2} := u_2;$$

$$R_{i3} := u_3 + 0.5u_6; R_{i4} := u_4;$$

$\epsilon_i :=$
$\epsilon_{i-1}; L_i := L_{i-1}$.

Step 6

OUTPUT(*AP*)
(AP approximates I to within ϵ.)

STOP.

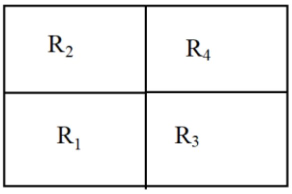

Figure 1

EXAMPLE

We illustrate the above algorithm to approximate the following double integral.

$$I = \iint\limits_{[1,3]\times[-1,3]} \frac{2x}{x^2 + y + 1} dA.$$

Here, we take a tolerance $\varepsilon = 0.0004, N = 4$. The above algorithm gives:

❖ When $L_i = 1$, $S_1 \approx 5.565190364$,
$S_2 \approx 5.526992146$,
$|S_1 - S_2| \approx 0.0382 > 15\varepsilon = 0.006$.

❖ When $L_i = 2$, we have

➢ On subrectangle $R_1 = [1,2] \times [-1,1]$, ("*the requirement is exceeded*")
$S_1 \approx 1.914412603$,
$S_2 \approx 1.909983661$,
$|S_1 - S_2| \approx 0.00443 > 15\varepsilon/4 = 0.0015$.

➢ On subrectangle $R_2 = [1,2] \times [1,3]$,
$S_1 \approx 1.133635232$,
$S_2 \approx 1.133629682$,
$|S_1 - S_2| \approx 5.55 \times 10^{-6} < 15\varepsilon/4 = 0.0015$.

➢ On subrectangle $R_3 = [2,3] \times [-1,1]$,
$S_1 \approx 1.396528503$,
$S_2 \approx 1.396452056$,
$|S_1 - S_2| \approx 7.64 \times 10^{-5} < 15\varepsilon/4 = 0.0015$.

➢ On subrectangle $R_4 = [2,3] \times [1,3]$,
$S_1 \approx 1.082415808$,
$S_2 \approx 1.082511495$
$|S_1 - S_2| \approx 9.57 \times 10^{-5} < 15\varepsilon/4 = 0.0015$.

❖ When $L_i = 3$, (*Continue subdividing* R_1 *into 4 smaller subrectangles*.)

➢ On subrectangle $R_{11} = [1,1.5] \times [-1,0]$,
$S_1 \approx 0.6201355326$,
$S_2 \approx 0.6197711285$,
$|S_1 - S_2| \approx 3.64 \times 10^{-4} < 15\varepsilon/16 = 3.75 \times 10^{-4}$.

➢ On subrectangle $R_{12} = [1,1.5] \times [0,1]$,
$S_1 \approx 0.4092542639$,
$S_2 \approx 0.4092359667$,
$|S_1 - S_2| \approx 1.83 \times 10^{-5} < 15\varepsilon/16 = 3.75 \times 10^{-4}$.

➢ On subrectangle $R_{13} = [1.5,2] \times [-1,0]$,
$S_1 \approx 0.4960067062$,
$S_2 \approx 0.4959783424$,
$|S_1 - S_2| \approx 2.84 \times 10^{-5} < 15\varepsilon/16 = 3.75 \times 10^{-4}$.

➢ On subrectangle $R_{14} = [1.5,2] \times [0.1]$,
$S_1 \approx 0.3845871583$,
$S_2 \approx 0.3845901215$,
$|S_1 - S_2| \approx 2.96 \times 10^{-6} < 15\varepsilon/16 = 3.75 \times 10^{-4}$.

Thus, the procedure succeeds at the level 3 of recursion, and it returns the approximation *AP* within the given tolerance $\varepsilon = 0.0004$ for the integral, where $AP = 5.522168792$. That means,

$$|AP - I| = |5.522168792 - I| < \varepsilon = 0.0004.$$

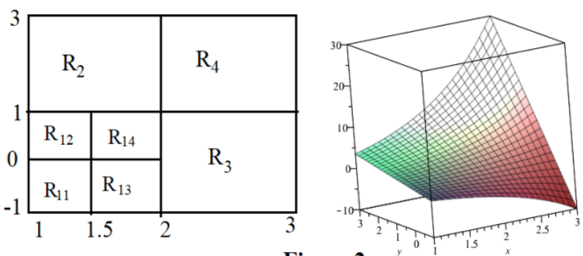

Figure 2

(Figure 2 illustrates the integrated region and the graph of the function.)

COMPARISON AND ANALYSIS OF THE OBTAINING RESULT OF THE EXAMPLE

For the comparison purpose, we list out results (represented in Table 1) produced by some other methods, which are quite popular. Those methods are Simpson's rule, Trapezoidal rule, and Midpoint rule (also known as the open Newton-Cotes of order 0), applied with different number of partitions. Table 1 shows the approximations of the double integral I produced by different methods applied on the number of subdivisions m=n from 1 to 9. Table also presents the corresponding error of these approximations. Here, m and n are the number of equal subintervals of the intervals $[1,3]$ and $[-1,3]$, respectively.

First, for this example, by a simple calculation, we can find the exact value of the double integral,

$$I = \int_{-1}^{3}\int_{1}^{3} \frac{2x}{x^2+y+1}\,dx\,dy$$
$$= \int_{-1}^{3} [\ln(y+10) - \ln(y+2)]\,dy$$
$$= 13\ln 13 - 9\ln 9 - 5\ln 5$$
$$\approx 5.5221308889.$$

The absolute error of the approximation produced by Adaptive quadrature is

$$|AP - I| \approx 0.000037903.$$

From the Table, we can see that obtaining a better approximation than AP requires applying Simpson's rule on at least 8 subintervals on each side of the rectangle $[1,3] \times [-1,3]$. For the other two methods, the requirement becomes infeasible when they are applied on at least 8 subintervals on each side of this rectangle. This feature clearly shows that Adaptive quadrature is much superior to the other three methods in reducing the number of calculation needed to obtain an approximation within a given tolerance.

SUMMARY

This method based on Simpson's and Composite Simpson's Rule, can also be constructed based on other method of the Numerical Integration, such as Trapezoidal Rule, Midpoint Rule. The idea used basically does not change. That is, we require the distribution of the error over integrated region with respect to the variation of the integrated function, large variation is corresponding to a smaller region, and small variation is corresponding to a bigger region. By the way, we often reach the requirement of the accuracy.

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Table 1. *The approximations for the integral in Example produced by different methods*

m=n	Midpoint rule	Error	Trapezoidal rule	Error	Simpson's rule	Error
1	5.555555557	0.033424668	5.584459984	0.062329095	5.565190365	0.043059476
2	5.541276320	0.019145431	5.498423800	0.023707089	5.526992147	0.004861258
3	5.531547187	0.009416298	5.507005307	0.015125582	5.523366556	0.001235667
4	5.527541247	0.005410358	5.512648185	0.009482704	5.522576888	0.000445999
5	5.525607164	0.003476275	5.515770460	0.006360429	5.522328257	0.000197368
6	5.524543773	0.002412884	5.517605037	0.004525852	5.522230854	0.000099965
7	5.523900866	0.001769977	5.518758245	0.003372644	5.522186671	0.000055782
8	5.523483800	0.001352911	5.519525437	0.002605452	5.522164430	0.000033541
9	5.523198278	0.001067389	5.520059824	0.002071065	5.522152185	0.000021296

TÓM TẮT

MỘT THUẬT TOÁN XẤP XỈ TÍCH PHÂN KÉP SỬ DỤNG PHƯƠNG PHÁP CẦU PHƯƠNG THÍCH ỨNG

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Trong các phương pháp tích phân số đối với trường hợp một biến, phương pháp Cầu phương Thích ứng đã rất phổ biến. Phương pháp này có ưu điểm vượt trội so với các phương pháp khác mà sử dụng các phép chia các khoảng bằng nhau là: nó làm giảm đi các phép tính giá trị hàm số, do đó giảm được các sai số làm tròn và tăng hiệu quả của phép xấp xỉ tích phân. Phương pháp này xem xét đến sự biến thiên của hàm số để điều chỉnh việc phân chia khoảng ban đầu thành các khoảng con có kích cỡ thích hợp tương ứng, trong đó khoảng con có kích cỡ nhỏ hơn sẽ tương ứng với sự biến thiên lớn hơn của hàm số trên khoảng đó. Việc phân chia này vẫn đảm bảo rằng sai số của phép xấp xỉ trong khoảng chấp nhận được cho trước. Nói cách khác, phương pháp này dựa trên ý tưởng phân phối đều sai số trên các khoảng bằng nhau. Bài báo này sẽ phát triển một thuật toán áp dụng phương pháp Cầu phương Thích ứng, dựa trên quy tắc Kết hợp Simpson, để xấp xỉ tích phân kép trên một miền tổng quát trong mặt phẳng. Ta sẽ chứng minh tính đúng đắn của thuật toán trong trường hợp này. Ngoài ra, bài báo cũng đưa ra phần mã giả của thuật toán và một số ví dụ tiêu biểu để minh họa cho thuật toán này. Các ví dụ này được thực hiện bằng cách sử dụng mã lệnh của Matlab.

Từ khóa: *tích phân kép, tích phân số, phương pháp Cầu phương Thích ứng, quy tắc Kết hợp Simpson, cầu phương.*

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