COMPARATIVE ANALYSIS OF POSITIVE REAL BALANCING AND MODAL TRUNCATION TECHNIQUES FOR MODEL REDUCTION IN ELECTRICAL AND ELECTRONIC SYSTEMS

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ABSTRACT

The complexity in modern electrical and electronic circuits has necessitated the need for efficient methods to manage behavior, simulate, and analyze them. Reduced-order modeling techniques are effective tools to address these challenges by simplifying systems while retaining essential characteristics. This research presents an analysis and comparison of two model reduction methods: Positive Real Balanced Truncation (PRR) and Modal Truncation Reduction (MTR). The characteristics of PRR and MTR are examined through an assessment of eighth-order electrical and electronic circuits. These techniques are implemented using Matlab to reduce the systems to second-order. Phase angle frequency response plots are generated to compare the original and reduced-order systems. Significant phase angle discrepancies between the reduced-order and original systems are observed at frequencies below 10^4 rad/s for both algorithms, beyond this frequency, the response matches that of the original system. Through a meticulous comparison of performance metrics, including absolute error, transient time, settling time, overshoot, and peak value, PRR demonstrates stability, precision during the transition phase, and more effective control capabilities compared to MTR.

PHÂN TÍCH, SO SÁNH KỸ THUẬT CHẶT CÂN BẰNG THỰC DƯƠNG VỚI CẮT NGẮN PHƯƠNG THỨC ĐỂ GIẢM BẬC CHO HỆ THỐNG ĐIỆN, ĐIỆN TỬ

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TỪ KHÓA

Giảm bậc mô hình
Bảo toàn sự thụ động
Cắt ngắn phương thức
Hệ thống điện
Mạch điện tử

TÓM TẮT

Độ phức tạp trong các mạch điện, điện tử hiện đại đã đặt ra nhu cầu cần phải có các phương pháp hiệu quả để quản lý hành vi, mô phỏng và phân tích. Các kỹ thuật giảm bậc mô hình là công cụ hiệu quả để giải quyết những thách thức này bằng cách đơn giản hóa hệ thống trong khi vẫn giữ lại các đặc tính cần thiết. Nghiên cứu này trình bày một phân tích, so sánh giữa hai phương pháp rút gọn mô hình: Chặt cân bằng thực dương (PRR) và Cắt ngắn phương thức (MTR). Đặc tính của PRR và MTR được xem xét thông qua đánh giá về mạch điện, điện tử bậc 8. Các kỹ thuật này được áp dụng bằng cách sử dụng Matlab để giảm hệ về bậc 2. Biểu đồ góc pha so theo tần số được tạo ra để so sánh giữa hệ gốc và hệ giảm bậc. Đáp ứng về góc pha rất sai khác giữa hệ rút gọn và hệ ban đầu, được quan sát ở các tần số dưới 10^4 rad/s với cả hai thuật toán, ngoài giá trị tần số này, đáp ứng trùng khớp với hệ gốc. Qua việc so sánh tỉ mỉ các chỉ số hiệu suất, bao gồm sai số tuyệt đối, thời gian tức thời, thời gian ổn định, độ quá điều chỉnh và giá trị đỉnh, PRR thể hiện tính ổn định, độ chính xác trong giai đoạn chuyển đổi và khả năng kiểm soát hiệu quả hơn so với MTR.

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1. Introduction

The inherent complexity of modern electrical and electronic systems has led to an array of pertinent issues that necessitate a strategic approach. Among the challenges that have emerged, complexity stands out as a driving factor. As systems become more intricate, managing and analyzing their behavior become increasingly convoluted. This complexity extends to the simulations required for system validation, which can be hindered by the sheer intricacy of these systems. Additionally, hardware limitations pose constraints on the degree of sophistication these systems can handle. This situation calls for techniques that can simplify these systems while preserving their essential characteristics. Moreover, temporal considerations play a pivotal role. Ensuring reliable real-time responses is crucial for a plethora of applications, yet the complexity of these systems can jeopardize this requirement. The intricacies of system dynamics might impede timely responses, leading to potential failures or suboptimal performance. Furthermore, simulation and analysis tasks are often hindered by the vast amount of data generated by these systems. Managing and processing this data can quickly become overwhelming and computationally demanding, prompting the need for more manageable representations.

To address the aforementioned challenges, the practice of model reduction and data dimensionality reduction proves to be essential. By reducing the complexity of system models, including their equations and variables, one can achieve a more streamlined representation that facilitates efficient analysis and simulation. This process leads to simplified models that retain the core behaviors and features of the original system, enabling quicker validation and exploration of different scenarios.

The properties of electrical, electronic, and power systems encompass stability, including a mix of stable and unstable sub-components, passivity (positive realness), and minimum phase behavior. To ensure that these physical characteristics are pre-served while reducing system complexity, model reduction approaches have been explored. Several research works address the challenge of model reduction while retaining stability and passivity properties. In [1], a model reduction technique based on positive real balanced truncation is introduced for second-order systems, ensuring stability and passivity preservation along with positive definiteness of matrices. A novel Structure-Preserving Reduced-order Interconnect Macromodeling (SPRIM) algorithm is proposed in [2], aiming to maintain input-output correlation matrix structure during model reduction. The need for model reduction in interconnect-dominated design due to shrinking device sizes and interconnect complexities is discussed in [3], highlighting the significance of spectral zeros for preserving stability and passivity. Moreover, power systems face challenges, and combining balanced and modal truncation approaches is explored in [4] to obtain a lower-order reduced model while accurately reproducing system behavior. Wind farm models are investigated in [5], where different linear model order reduction methods are applied, revealing merits and limitations. Experimental modal analysis and its truncation effects are examined in [6], emphasizing the influence of mode distribution on errors. Modal truncation is addressed from an optimization standpoint in [7], connecting dominant poles with optimal modal truncation. Lastly, the impact of modal truncation damping in piece-wise linear elastic structures is explored in [8], characterizing the behavior and limits of this phenomenon. These works collectively underscore the significance of model reduction techniques to address complexities in electrical, electronic, and power systems while preserving crucial system properties. In addition, the authors in [9] present the OHkNA algorithm for reducing the order of MIMO systems while preserving essential characteristics. They applied this algorithm to a 4-input, 4-output Microgrid MIMO system with an order of 14, reducing it to order 8 with minimal error. This approach proves valuable in the fields of control engineering, electrical systems, signal processing, and communication systems. Furthermore, in the article [10], the discussion centers on utilizing the H-Infinity balanced truncation (HBT) algorithm to simplify high-order electrical system models.

They successfully reduced a system with a degree of 66 to degrees 8 and 15, resulting in improved simulation time and efficiency in the analysis, design, and implementation of electrical networks. Nevertheless, it is crucial to note that both the OHkNA and HBT algorithms are primarily concerned with ensuring the stability of the original system and do not guarantee the preservation of its positive realness.

To validate the potential of preserving the physical properties of the original system, including stability and positive realness, the authors implement a model reduction algorithm based on balancing Gramians, specifically positive real balancing, and truncation techniques using either eigenvalues or dominant points. Subsequently, these two methods are applied to an electronic or electrical system, enabling the authors to draw conclusions and evaluate the advantages and limitations introduced by each approach.

2. Materials and Methods

2.1. Passivity-Preserving Positive Real Order Reduction Algorithm

The positive real balancing-based order reduction algorithm (PRR) involves balancing control and observation Gramians using positive real Riccati equations. This approach results in a reduced-order system obtained by truncating transformed system matrices based on positive real Gramian balancing. The solutions of these positive real Riccati equations yield a specific form, ensuring that the reduced-order system maintains the stability and passivity of the original system. The algorithm is designed to work with an asymptotically stable system, ensuring the preservation of passivity and meeting certain conditions, while describing the system using state space matrices. The desired outcome is a reduced-order system of the specified order. The PRR algorithm is described as follows [4]:

Input: An asymptotically stable system, passive (positive real), described by four matrices in state space representation, (**A, B, C, D**) with an order of *n*, the desired reduced order *r*.

- Step 1: Solve two positive real Riccati equations:

$$\mathbf{A}\mathbf{X}_p + \mathbf{X}_p\mathbf{A}^T + \left(\mathbf{X}_p\mathbf{C}^T - \mathbf{B}\right)\left(\mathbf{D} + \mathbf{D}^T\right)^{-1}\left(\mathbf{X}_p\mathbf{C}^T - \mathbf{B}\right)^T = \mathbf{0} \quad (1)$$

$$\mathbf{A}^T\mathbf{Y}_p + \mathbf{Y}_p\mathbf{A} + \left(\mathbf{C}^T - \mathbf{Y}_p\mathbf{B}\right)\left(\mathbf{D} + \mathbf{D}^T\right)^{-1}\left(\mathbf{C}^T - \mathbf{Y}_p\mathbf{B}\right)^T = \mathbf{0} \quad (2)$$

- Step 2: Cholesky decomposition:

$$\mathbf{X}_p = \mathbf{R}_p\mathbf{R}_p^{*} \quad (3)$$

$$\mathbf{Y}_p = \mathbf{L}_p\mathbf{L}_p^{*} \quad (4)$$

- Step 3: Singular Value Decomposition:

$$\mathbf{L}_p^T\mathbf{R}_p = \mathbf{U}_p\mathbf{\Sigma}_p\mathbf{V}_p^{*} \quad (5)$$

- Step 4: Calculate the transformation matrix:

$$\mathbf{T}_p = \mathbf{R}_p\mathbf{V}_p\mathbf{\Sigma}_p^{-\frac{1}{2}} \quad (6)$$

$$\mathbf{T}_p^{-1} = \mathbf{\Sigma}_p^{-\frac{1}{2}}\mathbf{U}_p^{*}\mathbf{L}^{*}_p \quad (7)$$

- Step 5: Convert to a balanced system:

$$\mathbf{T}_p^{-1}\mathbf{A}\mathbf{T} = \mathbf{A}_p;\ \mathbf{T}_p^{-1}\mathbf{B} = \mathbf{B}_p;\ \mathbf{C}\mathbf{T}_p = \mathbf{C}_p \quad (8)$$

- Step 6: Choose the order r to reduce *r (r < n)*

Output: A reduced-order system of order *r*, achieved by eliminating the elements of the balancing matrices determined from (7) in the corresponding rows and columns starting from the position *n-r* onwards.

2.2. Model order reduction based on Modal Truncation

The fundamental concept of Modal Truncation (MTR) involves the projection of a linear time-invariant system onto an invariant subspace that aligns with specific constraints, such as poles or eigenvalues. In this approach, emphasis is placed on retaining eigenvalues characterized by non-negative or slightly negative real components, which exert a dominant influence on the system's long-term behavior. Meanwhile, eigenvalues with rapid decay and less significant contributions, typically associated with large negative real components, are neglected. The MTR algorithm rooted in eigenvalues can be outlined as follows [7]:

Input: Given an asymptotically stable system (**A, B, C, D**) with a minimal order of n as described by equation:

- Step 1: Compute the state transformation matrices **T** and its inverse $\mathbf{T^{-1}}$ to satisfy the relationship:

$$\mathbf{T^{-1}AT} = \begin{bmatrix} \eta_1 & & \\ & \ddots & \\ & & \eta_n \end{bmatrix};\ \mathbf{T^{-1}B} = \begin{bmatrix} \mathbf{B_1} \\ \mathbf{B_2} \end{bmatrix}; \mathbf{CT} = \begin{bmatrix} \mathbf{C_1} & \mathbf{C_2} \end{bmatrix} \tag{9}$$

where $\eta_1, \eta_2, ..., \eta_n, \ldots$ represent the singular values of the system arranged in ascending order $0 > \mathrm{Re}(\eta_1)\ldots\mathrm{Re}(\eta_2)\ldots\ldots\mathrm{Re}(\eta_n)$.

- Step 2: Choose the desired reduced order r.
- Step 3: Determine the matrices **U** and **V** that fulfill:

$$\mathbf{U} = \mathbf{T}\begin{bmatrix} \mathbf{I}_n \\ 0 \end{bmatrix} = \mathbf{T}(:,1:r) \tag{10}$$

$$\mathbf{V} = \mathbf{T}^{-T}\begin{bmatrix} \mathbf{I_r} \\ 0 \end{bmatrix} = \left(\mathbf{T}^{-1}(1:r,:)\right)^T \tag{11}$$

Output: The resulting reduced-order system with asymptotic stability:

$$\mathbf{A_r} = \mathbf{V^{*}\ AU};\ \mathbf{B_r} = \mathbf{V^{*}\ B};\ \mathbf{C_r} = \mathbf{CU};\ \mathbf{D_r} = \mathbf{D} \tag{12}$$

3. Results and Discussion

Considering an electrical and electronic circuit as shown in Figure 1, we have the total impedance $Z_1(s)$, and the transfer function $H_1(s)$ and the transfer function:

$$Z_1(s) = \frac{R_0LCs^2 + (R_0RC + L)s + R_0 + R}{LCs^2 + RCs + 1} \tag{13}$$

$$H_1(s) = \frac{I(s)}{U(s)} = \frac{LCs^2 + RCs + 1}{R_0LCs^2 + (R_0RC + L)s + R_0 + R} \tag{14}$$

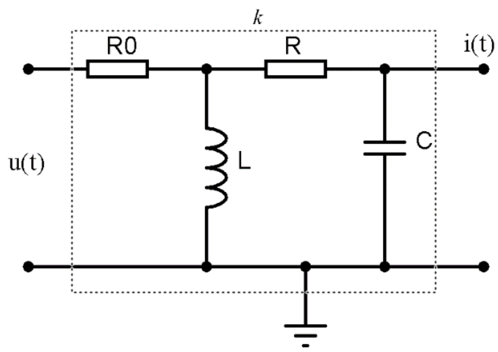

Figure 1. *Phase Response Comparison for Reduced-Order Systems using PRR and MTR.*

This circuit is of second order, consisting of four cascaded subsections, resulting in an overall system order of 8. To convert the system's transfer function into a state-space representation, which includes matrices **A**, **B**, **C**, and **D** (in Matlab, this can be accomplished using the command: *[A, B, C, D] = tf2ss(num, den)*, where *num* and *den* respectively represent the numerator and denominator of the system's transfer function).

We proceed to implement the PRR and MTR algorithms using Matlab to reduce the order of the original system to any order r ($r < 8$), here the authors choose $r = 2$. We then create phase angle versus frequency plots for both methods, depicting the comparison between the original system and the reduced-order systems using each technique, as shown in Figure 2.

From Figure 2, it is evident that when employing the PRR and MTR techniques to reduce the order of the original system to second-order, both methods yield phase response discrepancies in frequency ranges below 10^4 rad/s. Beyond this frequency value, the phase response of the reduced-order systems obtained from these two techniques closely matches the original system.

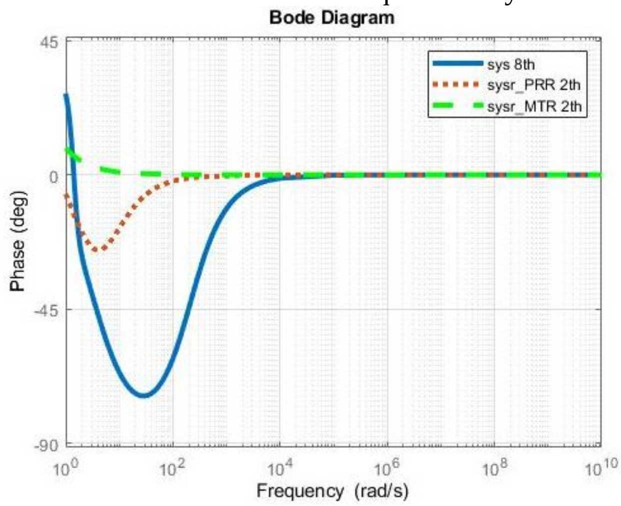

Figure 2. *Phase Response Comparison for Reduced-Order Systems using PRR and MTR.*

The PRR and MTR techniques exhibit similar behavior in terms of phase response alteration for the reduced-order systems when compared to the original higher-order system. The discrepancies observed in the lower frequency range might indicate limitations or inaccuracies introduced by the reduction methods in capturing certain dynamic aspects of the original system.

We conducted pulse response simulations on Matlab for the second-order reduced system using PRR and MTR in the time domain, obtaining results as shown in Table 1.

Table 1. *Comparison of System Performance Metrics when reducing model using PRR và MTR*

Comparison Aspect	PRR	MTR
Absolute error	0.47532	0.77793
Transient Time (s)	18.46775	20.36918
Settling Time (s)	19.48650	28.31976
Settling Min	0.14142	0.00178
Settling Max	0.32050	0.01000
Overshoot (%)	126.62909	460.40133
Peak	0.32050	0.01000

Table 1 summarizes a comparison between PRR and MTR model reduction methods. It includes key performance metrics such as absolute error, transient time, settling time, overshoot, and peak values. The comparison table provides a comprehensive overview of how PRR and MTR fare across different metrics. The PRR technique yields a lower norm error of 0.47532 compared to MTR's norm error of 0.77793. This suggests that PRR achieves a closer match between the reduced-order model and the original system dynamics.

In terms of transient time, PRR demonstrates a shorter time of 18.46775 seconds, while MTR exhibits a slightly longer transient time of 20.36918 seconds. This indi-cates that PRR captures the system's initial response more accurately. Similarly, PRR's settling time of 19.48650 seconds is notably smaller than MTR's 28.31976 seconds, suggesting faster convergence to steady-state behavior.

The settling minima and maxima reflect the smallest and largest values during the settling period. PRR outperforms MTR in both categories with values of 0.14142 and 0.32050, respectively, compared to MTR's 0.00178 and 0.01000. This underscores PRR's ability to maintain stability and accuracy during transient phases.

The overshoot metric signifies the peak value above the final steady-state value during the response. Surprisingly, PRR exhibits a much lower overshoot of 126.62909 compared to MTR's significantly higher 460.40133. This result suggests that PRR achieves better control over the overshoot phenomenon, thereby preventing exces-sive deviations.

Comparing the peak values of PRR and MTR provides insights into their perfor-mance. PRR's peak value of 0.32050 indicates better preservation of system dynamics compared to MTR's value of 0.01000. PRR captures and reproduces peak responses accurately, suggesting its superior ability to retain critical dynamic features. In contrast, MTR's lower peak value implies limitations in accurately representing transient behaviors.

4. Conclusion

In this study, we addressed the challenges posed by the complexity of modern electrical and electronic systems through model reduction techniques. The complexities of these systems, stemming from their intricate behavior and simulation demands, call for approaches that can simplify the systems while retaining their essential characteristics. Model reduction offers a strategic solution by streamlining system models and facilitating efficient analysis and simulation.

The comparison between the PRR and MTR model reduction techniques revealed valuable insights. Both techniques demonstrated their ability to reduce the order of an 8th-order electrical and electronic circuit while preserving key system properties. However, the comparison table elucidates nuanced differences in their performance.

The PRR technique achieved a lower norm error of 0.47532 compared to MTR's norm error of 0.77793, indicating PRR's superior ability to capture the original sys-tem's dynamics. Additionally, PRR exhibited shorter transient and settling times (18.46775 s and 19.48650 s, respectively) compared to MTR (20.36918 s and 28.31976 s), suggesting faster convergence to steady-state behavior and more accu-rate initial response representation.

The settling minima and maxima metrics further emphasized PRR's superiority, with values of 0.14142 and 0.32050, respectively, outperforming MTR's values of 0.00178 and 0.01000. PRR's better stability and accuracy during transient phases contribute to its overall effectiveness. Furthermore, PRR effectively controlled over-shoot, exhibiting a significantly lower value of 126.62909 compared to MTR's 460.40133, indicating its ability to prevent excessive deviations.

Comparing peak values highlighted PRR's strength in preserving system dynamics, as reflected by its peak value of 0.32050, while MTR exhibited a value of 0.01000. This difference underscores PRR's superior ability to accurately reproduce critical dynamic features, revealing potential limitations of MTR in accurately representing transient behaviors.

In conclusion, both PRR and MTR offer valuable methods for reducing the order of complex electrical and electronic systems while preserving crucial system proper-ties. However, the comprehensive analysis presented in this study demonstrates PRR's overall superiority in terms of accuracy, stability, and dynamic feature preser-vation. The choice between these techniques should consider specific system requirements and desired performance metrics.

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