STOCHASTIC POSITIVE-REAL BALANCED TRUNCATION: A NOVEL MODEL ORDER REDUCTION TECHNIQUE

Nguyen Thuy Dung[1], Nguyen Thanh Tung[1*], Vu Ngoc Kien[2], Dao Huy Du[2]
[1]*TNU - University of Information and Communication Technology*
[2]*TNU - University of Technology*

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In the context of the digital revolution, the development of low-order model reduction techniques for electrical and electronic circuits is imperative. In this paper, we introduce a novel algorithm called Stochastic Positive-Real Balanced Truncation (SPBT) based on the combination of two techniques: Positive-Real Balanced Reduction (PRR) and Stochastic Balanced Reduction (SBR) to reduce the model order. From the experimental results and comparisons of SPBT with PRR and SBR methods on electronic circuit models, it is evident that SPBT not only preserves the stability, passivity, and minimum phase properties of the original system but also provides higher performance and accuracy compared to PRR and SBR. With low errors and matching responses between the reduced-order system and the original system when applying SPBT in the time and frequency domains, this method holds promise as an effective tool for reducing complexity while maintaining the essential physical characteristics of the system.

CẮT NGẮN CÂN BẰNG NGẪU NHIÊN-THỰC DƯƠNG: MỘT KỸ THUẬT MỚI TRONG GIẢM BẬC MÔ HÌNH

Nguyễn Thuỳ Dung[1], Nguyễn Thanh Tùng[1*], Vũ Ngọc Kiên[2], Đào Huy Du[2]
[1]*Trường Đại học Công nghệ Thông tin và Truyền thông – ĐH Thái Nguyên*
[2]*Trường Đại học Kỹ thuật Công nghiệp – ĐH Thái Nguyên*

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TỪ KHÓA

Giảm bậc mô hình
Thực-dương
Cực tiểu pha
Cắt ngắn cân bằng ngẫu nhiên
Mạch điện

Trong bối cảnh của cuộc cách mạng số hóa, việc phát triển các kỹ thuật giảm bậc mô hình cho mạch điện, điện tử là vấn đề cấp thiết. Trong bài báo này, chúng tôi giới thiệu một thuật toán mới gọi là Stochastic Positive-Real Balanced Truncation (SPBT) dựa trên sự kết hợp của hai kỹ thuật Cắt ngắn cân bằng thực dương (PRR) và Cắt ngắn cân bằng ngẫu nhiên (SBR) để giảm bậc mô hình. Từ kết quả thử nghiệm và so sánh SPBT với các phương pháp PRR và SBR trên mô hình mạch điện tử, cho thấy SPBT không chỉ bảo toàn tính ổn định, sự thụ động và cực tiểu pha của hệ gốc mà còn mang lại hiệu suất và độ chính xác cao hơn so với PRR và SBR. Với sai số nhỏ, sự trùng khớp về đáp ứng giữa hệ giảm bậc và hệ gốc khi áp dụng SPBT trong miền thời gian và miền tần số, phương pháp này hứa hẹn là một công cụ hiệu quả trong việc giảm độ phức tạp mà vẫn duy trì tính chất vật lý quan trọng của hệ thống.

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[*] Corresponding author. *Email: nttung@ictu.edu.vn*

1. Introduction

The digital age, characterized by Industry 4.0, introduces a plethora of keywords such as Intelligent Robots, IoT, AI, and more. Designing advanced electronic circuits, compact yet high-performing, remains a challenge. Simulation expedites this process, converting output requirements into solvable variables. An approach involving model order reduction (MOR) and high-dimensional data reduction aims to expedite simula-tions, enhance computations, and reduce hardware stress.

Electrical circuits' passive, positive-real, and minimum-phase traits are crucial. MOR methods maintaining these properties are explored. The article [1] presents a method for electronic circuits that utilizes Krylov and Lyapunov techniques to ensure stability and passivity while incorporating adaptive PI control. The document [2] concentrates on the PH model of RLC networks, preserving passivity through ε-embedding and time-matching. The authors in [3] employ MOR (Model Order Reduction) for on-chip interconnects, using spectral zeros to maintain stability. Various MOR techniques that uphold stability are discussed. The scientific work [4] reduces the order of unstable power plant systems while retaining essential properties. The research [5] introduces three MOR methods for wind farms and tests them on practical-sized farms. The article [6] optimizes Automatic Generation Control (AGC) using PI-PD control, enhancing transient and steady-state performance. The document [7] suggests a frequency-weighted Gramians-based technique that excels in delivering steady and accurate results. The group of authors in [8] reduces the order of power systems while preserving their fundamental characteristics. However, these methods do not adequately address minimum-phase preservation. The paper [9] reduces complex interconnections using balanced stochastic truncation, ensuring an accurate frequency response. The research [10] evaluates model reduction strategies for grid-tied inverters, highlighting balanced decidualization as suitable for a trade-off between accuracy and complexity. Nevertheless, these methods primarily focus on minimum-phase characteristics and neglect the passive (positive-real) attribute.

In the ever-evolving landscape of technology, maintaining the equilibrium be-tween performance and complexity remains a continuous challenge. Based on the aforementioned survey, to develop a method capable of model reduction while re-taining the physical characteristics of the original system (in electrical and electronic circuit diagrams), including passivity (positive-real nature), stability, and minimum-phase behavior in the reduced model, the authors propose an algorithm to satisfy these requirements.

To verify the newly proposed technique, the authors implement the algorithm using Matlab and apply it to an electronic circuit model. Subsequently, a thorough comparison, evaluation, and analysis of the proposed approach are conducted to assess its effectiveness.

2. Materials and Methods

A standard linear time-invariant (LTI) state-space system is a collection of matrix differential equations represented as (1):

$$G(s):\begin{cases}\dot{x}(t)=\mathbf{A}x(t)+\mathbf{B}u(t)\\ y(t)=\mathbf{C}x(t)+\mathbf{D}u(t)\end{cases} \tag{1}$$

Where x(t) is the vector of n state variables, and represents the derivative of x(t) with respect to the time variable t. The matrices (**A**, **B**, **C**, **D**) are the state-space matrices that define the dynamic behavior of the system. Vector *u(t)* represents the excitation at the inputs, while y(t) represents the outputs.

This segment introduces an innovative algorithm designed for systems exhibiting both positive (passive) and minimum-phase attributes. This technique facilitates the reduction of the order of stable systems, akin to methods reliant on Gramians. Importantly, it preserves the inherent passive (positive) and minimum-phase characteristics of the original system. The approach involves the utilization of the positive-definite Riccati equation (2) and Riccati equation (3), subsequently transforming them into a balanced realization. Referred to as the Stochastic Positive-Real Balanced Truncation Realization Algorithm (SPBT), this method represents a significant advancement in the realm of model reduction techniques.

Algorithm. Stochastic Positive-Real Balanced Truncation Algorithm (SPBT):

Input: The Linear Time-Invariant (LTI) system G(s) (1) exhibits attributes of asymptotic stability, minimality, squareness, and no singularity, along with being minimum phase and possessing positive real (passive) characteristics, all with an order of *n*. The aim is to diminish the system's order to *r* (where *r* < *n*).

- Step 1: Solve Lyapunov equation (1), solve two Riccati equations (2) and (3), where the matrices (**A**, **B**, **C**, **D**) are the state-space matrices that define the dynamic behavior of the system:

$$\mathbf{A^T Y_b + \left(C - B_b^{*} Y_b\right)^T \left(DD^T\right)^{-1} \left(C - B_b^{*} Y_b\right) = -Y_b A} \quad (1)$$

$$\mathbf{AX_p + \left(B - X_p C^T\right)\left(D + D^T\right)^{-1}\left(B - X_p C^T\right)^T = -X_p A^T} \quad (2)$$

$$\mathbf{A^T Y_b + \left(C - B_b^{*} Y_b\right)^T \left(DD^T\right)^{-1} \left(C - B_b^{*} Y_b\right) = -Y_b A} \quad (3)$$

where $\mathbf{B_b := PC^T + BD^T}$, **P** and $\mathbf{Y_b}$ are respectively the Gramian of control and the Gramian of observation for the balanced stochastic original system, and $\mathbf{X_p}$ is the Gramian of positive real control for the original system.

- Step 2: Cholesky decomposition:

$$\mathbf{X_p = R_p R_p^{*}} \quad (4)$$

$$\mathbf{Y_b = L_b L_b^{*}} \quad (5)$$

- Step 3: Singular Value Decomposition:

$$\mathbf{L_b^{*} R_p = U_{SP} \Sigma_{SP} V_{SP}^{*}} \quad (6)$$

- Step 4: Calculate the transformation matrix:

$$\mathbf{T = R_p V_{SP} \Sigma_{SP}^{-\frac{1}{2}}} \quad (7)$$

$$\mathbf{T^{-1} = \Sigma_{SP}^{-\frac{1}{2}} U_{SP}^{*} L_b^{*}} \quad (8)$$

- Step 5: Convert to a balanced system:

$$\mathbf{T^{-1}AT = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; T^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}; CT = \begin{bmatrix} C_1 & C_2 \end{bmatrix}} \quad (9)$$

where: $A_{11} \in R^{r\times r}, A_{12} \in R^{r\times(n-r)}, A_{21} \in R^{(n-r)\times r}, A_{22} \in R^{(n-r)\times(n-r)}, B_1 \in R^{r\times m}, B_2 \in R^{(n-r)\times m}, C_1 \in R^{p\times r}, C_2 \in R^{p\times(n-r)}$

- Step 6: Choose the order r to reduce *r (r < n)*

Output: Reduced-order system with order *r* preserves both positive real (passive) and minimum phase properties: $G_r(s) : \left(\mathbf{A_r, B_r, C_r, D_r}\right) = \mathbf{(A_{11}, B_1, C_1, D)}$.

3. Results and Discussion

Consider an electronic circuit described by four matrices A, B, C, and D in a state-space representation as follows:

$$\mathbf{A}=\begin{bmatrix}0&0&0&0&2&0&0&0\\0&0&0&0&0&2&0&0\\0&0&0&0&0&0&2&0\\0&0&0&0&0&0&0&2\\-1&0&0&0&-4&2&0&0\\0&-1&0&0&2&-6&4&0\\0&0&-1&0&0&4&-6&2\\0&0&0&-1&0&0&2&-4\end{bmatrix},\mathbf{B}=\begin{bmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{bmatrix},\mathbf{C}^{T}=\begin{bmatrix}0\\0\\0\\0\\2\\0\\0\\0\end{bmatrix},\mathbf{D}=0.01$$

We proceed to implement the PRR, SBR, and the proposed SPBT algorithms in MATLAB. Subsequently, we apply each method to gradually reduce the order of the original system from order 8 to order 1. This process yields a plot depicting the absolute errors between the original and reduced-order systems as shown in Figure 1, with the errors listed in Table 1.

Table 1 clearly illustrates the performance of PRBT, SBR, and PB methods across different reduced orders. PB consistently outperforms the other methods in terms of accuracy. The increase in PRBT error for higher r suggests a limitation in maintaining accuracy with aggressive reduction. SBR maintains consistent but higher errors, im-plying a potential trade-off between accuracy and dimensionality reduction. PB's ability to consistently achieve low error values across various dimensions reaffirms its effectiveness as a reduction technique.

Table 1. *Comparison of Reduction Methods (Error Values).*

Reduced Order (r)	PRR Error	SBR Error	SPBT Error
1	0.78339	0.77802	0.49956
2	0.60391	0.77579	0.27822
3	0.60454	0.81758	0.22726
4	0.74246	0.81603	0.03888
5	0.45217	0.74070	0.01299
6	1.25389	0.75626	0.00191
7	0.81324	0.66944	0.00050

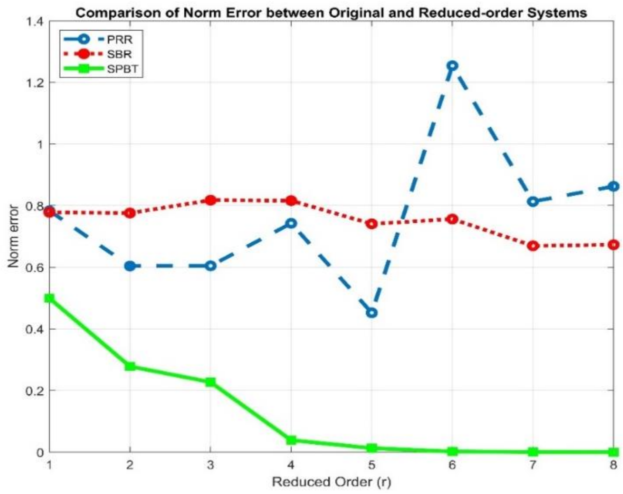

Figure 1. *Comparing Absolute Errors in Model Reduction Techniques among PRR, SBR, SPBT*

From Figure 1 and Table 1, it is observed that when employing the PRR method, the maximum error occurs when reducing the system to order 6, yielding an approxi-mate error of 1.25389. Conversely, the minimum error occurs when reducing the system to order 5, resulting in an approximate error of 0.45217. For the SBR tech-nique, the maximum error arises when reducing the system to order 3 with an error of 0.81758, while the minimum error occurs at order 7 with an error of 0.66944. Mean-while, applying the SPBT algorithm to reduce the order leads to a maximum error of approximately 0.49956 at order 1 and a minimum error of around 0.00050 at order 7.

- The PRR method shows varying results across different values of r. As *r* increases, the error initially decreases, suggesting that the reduced model captures more dynamics of the original system. However, after reaching a certain point, the error starts increasing, indicating that further reduction leads to loss of system information. For higher values of r, the PRR method performs less effectively in maintaining accuracy.

- The SBR method demonstrates relatively consistent error values across different r. The error values are generally higher compared to PRR for the range of *r* considered. This suggests that SBR may not capture all the relevant dynamics of the original system as effectively as PRR in this case.

- The SPBT method consistently outperforms both PRR and SBR across all values of r. The error values decrease rapidly with increasing *r*, indicating that SPBT is able to achieve accurate reduced-order models even with relatively low dimensions. This suggests that combining the strengths of PRR and SBR through SPBT results in a powerful reduction method.

- PRR appears to perform well initially but struggles to maintain accuracy for higher values of r. It may be suitable for moderate reduction but loses accuracy with aggressive reduction.

- SBR shows consistent but relatively higher error values, indicating that it might not capture the system dynamics as effectively as other methods.

- SPBT consistently yields the lowest error values, showcasing its ability to produce accurate reduced-order models across various dimensions.

Figure 2 and Figure 3 correspondingly display the step response and Bode frequency response (Phase, Magnitude) between the original system and the reduced-order systems of order 4 using the PRR, SBR, and the newly proposed SPBT techniques.

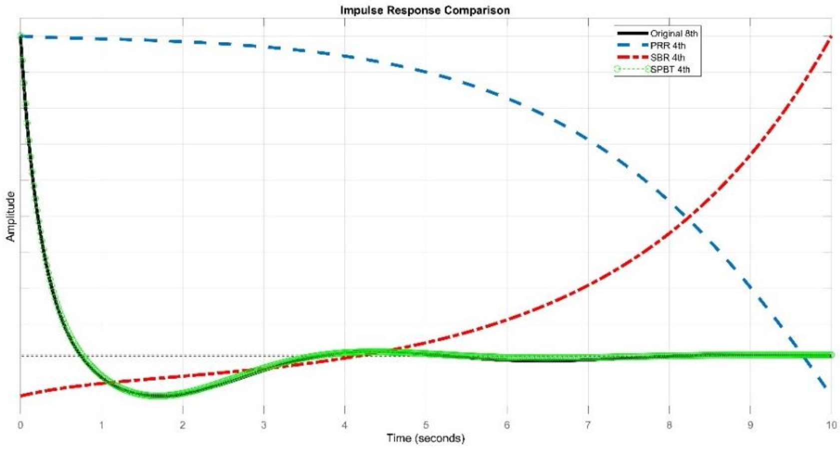

Figure 2. *Impulse response Comparison between Reduced-Order Systems using PRR, SBR, SPBT and the Original System*

From Figure 2, the following observations can be made:

- The reduced-order systems of order 4 using the PRR and SBR methods significantly differ from the original system in the step response. In contrast, the SPBT technique yields a step response that closely matches the original system's response across the entire time domain.

- Considering the effectiveness of reducing the original system to a fourth-order model, it can be inferred that the fourth-order reduced model obtained via the SPBT algorithm can be a substitute for the eighth-order original system in the time domain.

From Figure 3, the following insights can be drawn:

- The reduced-order systems of order 4 using the PRR and SBR methods exhibit notable discrepancies in the Phase and Magnitude responses over the frequency range approximately from 10^{-4} rad/s to 10^{4} rad/s, whereas beyond this range, these responses closely adhere to the original system. Therefore, a fourth-order model can potentially replace the original system for frequencies smaller than 10^{-4} rad/s and greater than 10^{4} rad/s in the frequency domain. In this context, if maintaining passive behavior is a priority, PRR is a suitable choice; however, if preserving minimal phase behavior is essential, the SBR technique is preferable.
- In contrast, the SPBT algorithm ensures a frequency response that matches the original system's response across the entire frequency domain. Considering the efficacy of reducing the original system to a fourth-order model, it can be inferred that the fourth-order reduced model obtained via the SPBT algorithm can replace the eighth-order original system throughout the frequency domain.

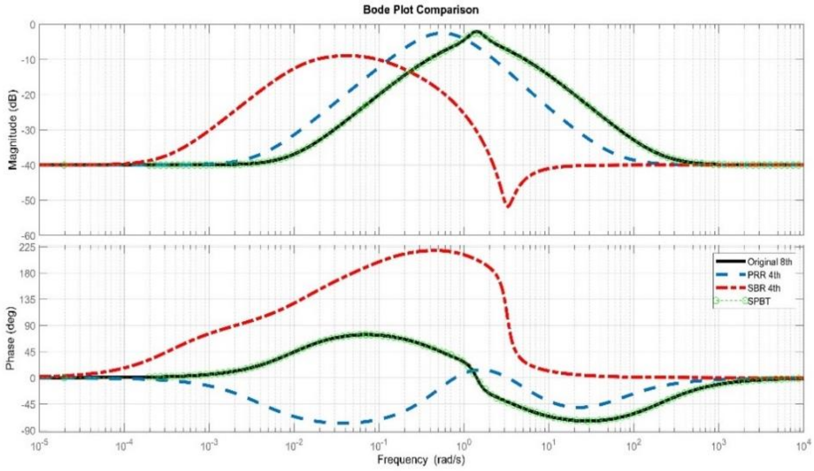

Figure 3. *Bode Plot Comparison between Reduced-Order Systems using PRR, SBR, SPBT and the Original System*

The reduced-order system achieved through the novel SPBT algorithm proposed by the authors not only exhibits smaller errors compared to the PRR and SBR methods but also closely adheres to the original system in both step response and frequency response. Furthermore, the SPBT algorithm excels in reduction tasks by simultaneously preserving passive behavior and maintaining minimal phase behavior.

4. Conclusion

In the rapidly evolving landscape of technology, the challenge of balancing performance and complexity in the realm of model reduction techniques for electrical and electronic circuits persists. This study has explored the development and evaluation of an innovative algorithm, the Stochastic Positive-Real Balanced Truncation (SPBT), designed to address this challenge by significantly advancing the field of model reduction.

The proposed SPBT algorithm offers a powerful solution for reducing the order of stable systems while preserving essential physical characteristics, including passivity, stability, and minimum-phase behavior. This algorithm utilizes a combination of the positive-definite Riccati equation and balanced realization, resulting in a model reduction technique that excels in both accuracy and dimensionality reduction. Our results and discussions demonstrate the effectiveness of the SPBT algorithm in achieving these goals. Comparing SPBT with traditional reduction methods, such as PRR and SBR, reveals its superiority in terms of accuracy and fidelity to the original system. It consistently produces reduced-order systems that closely match the step and frequency responses of the original system, even in cases of aggressive dimensionality reduction. This suggests that the SPBT algorithm can serve as an efficient replacement for the original system, offering significant benefits in terms of computational efficiency and resource conservation.

In the ever-advancing world of technology, where the demand for high-performance electronic circuits remains constant, the SPBT algorithm emerges as a valuable tool. Its ability to reduce the order of complex systems while preserving their essential attributes is a testament to its effectiveness in the design and simulation of electronic circuits. This algorithm contributes to the ongoing efforts to bridge the gap between system complexity and computational efficiency, ultimately benefiting various applications in the field of electronics and beyond. As technology continues to evolve, it is essential to adapt and innovate, and the SPBT algorithm is a promising step in the right direction.

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