RECOMMENDATION OF DEFLECTION CALCULATION FOR A REINFORCED CONCRETE BEAM SUBJECTED TO SHORT-TERM LOADS ACCORDING TO VIETNAMESE STANDARD TCVN 5574:2018: A COMPARISON WITH FINITE ELEMENT MODELING

Tran Thanh Binh*, Nguyen Quang Tung, Trinh Quang Thinh, Vuong Le Thang, Truong Hoai Chinh The University of Danang - University of Science and Technology

ARTICLE INFO		ABSTRACT
Received:	21/3/2024	This study focuses on evaluating short-term deflections of reinforced
Revised:	23/5/2024	concrete beams according to the Vietnamese Standard on the Design of Concrete and Reinforced Concrete Structures (TCVN 5574:2018) by
Published:	24/5/2024	comparing them with simulated results obtained from Abaqus software.
		The aim is to propose recommendations to enhance safety in calculating
KEYWORDS		deflections according to TCVN 5574:2018. The Concrete Damaged
1121 (10112)		-Plasticity model is defined in the Abaqus software to account for complex
TCVN 5574:2018		behaviors of concrete, while in the TCVN 5574:2018, bilinear or trilinear
Deflection		stress-strain curves are used to facilitate practical calculations. It is found
Reinforced Concrete		that values of deflections calculated according to TCVN 5574:2018 could
		be underestimated during the non-cracking stage of concrete and
Concrete Damage Plan	sticity	overestimated at higher levels of loads before reaching the yielding point of
Finite Element Model	ing	steel with differences varied between 15% and 20%. By comparing results
	-	from practical standards and complex simulations, this research contributes
		to the enhancement of design methodologies and structural safety.

KHUYẾN NGHỊ KHI TÍNH TOÁN ĐỘ VÕNG NGẮN HẠN CỦA CẦU KIỆN BÊ TÔNG CÓT THÉP THEO TIÊU CHUẨN TCVN 5574:2018 QUA VIỆC SO SÁNH VỚI MÔ PHỎNG PHẦN TỬ HỮU HẠN

Trần Thanh Bình*, Nguyễn Quang Tùng, Trịnh Quang Thịnh, Vương Lê Thắng, Trương Hoài Chính Trường Đại học Bách khoa - ĐH Đà Nẵng

THÔNG TIN BÀI I	BÁO	TÓM TẮT
Ngày nhận bài: 2	1/3/2024	Nghiên cứu này tập trung vào việc đánh giá độ võng ngắn hạn của dầm bê
Ngày hoàn thiện: 2	3/5/2024	tông cốt thép được tính theo Tiêu chuẩn Việt Nam về thiết kế kết cấu bê tông và bê tông cốt thép (TCVN 5574:2018) bằng cách so sánh với kết quả
Ngày đăng: 2	4/5/2024	từ phân mêm Abaqus. Mục tiêu là đề xuất các khuyên cáo nhằm nâng cao
		độ an toàn cho việc tính độ võng bằng TCVN 5574:2018. Mô hình kể đến
TỪ KHÓA		hư hại dẻo của bê tông được thiết lập trong phần mềm Abaqus nhằm kể đến
		ứng xử phức tạp của vật liệu bê tông, trong khi đó ở TCVN 5574:2018,
TCVN 5574:2018		quan hệ ứng suất - biến dạng hai hoặc ba đoạn thẳng của bê tông được sử
Độ võng		dụng để thuận tiện trong tính toán thực hành. Kết quả cho thấy giá trị độ
Bê tông cốt thép		võng tính toán theo TCVN 5574:2018 cho giá trị thấp hơn kết quả mô
		phỏng trong giai đoạn bê tông chưa nứt và cho giá trị cao hơn với các cấp
Hư hại đẻo của bê tông		tải trọng lớn hơn trước khi đạt đến giới hạn chảy của thép với chênh lệch
Mô phỏng phần tử hữu	hạn	dao động từ 15% đến 20%. Bằng cách so sánh kết quả từ tiêu chuẩn thực
		hành và mô phỏng phức tạp, nghiên cứu này đóng góp vào việc cải thiện
		các phương pháp thiết kế và nâng cao độ an toàn của kết cấu.

DOI: https://doi.org/10.34238/tnu-jst.9941

http://jst.tnu.edu.vn

229(06): 178 - 186

^{*} Corresponding author. Email: ttbinh1@dut.udn.vn

1. Introduction

During the design process of construction projects, prediction and control of deflection are crucial tasks to ensure normal level of structural safety and serviceability. The actual Vietnamese Standard on the Design of Concrete and Reinforced Concrete Structures (TCVN 5574:2018) [1] has made significant updates in calculation of deflection in reinforced concrete (RC) structures. These improvements aim to include material behaviors by considering bilinear or trilinear stress-strain curves and to account for the effect of creep in concrete by introducing a degradation factor in its Young's Modulus. These assumptions could facilitate practical calculations; however, they cannot accurately describe the complex real behaviors of materials, which require a more advanced material model.

There have been several studies related to the deflection of RC components according to TCVN 5574:2018. N.L. Nguyen et al. [2] applied the bilinear/trilinear stress-strain curve model to predict the ultimate bending moment of RC beams using the load-deflection diagrams obtained from experiments. H.A.T. Nguyen et al. [3] proposed a methodology to establish the relationship between the bending moment and curvature of the beam with consideration of nonlinear stress-strain relationship of concrete and steel defined in the TCVN 5574:2018. Recently, M.T. Phan et al. [4] used the stress-strain curves of materials specified in TCVN 5574:2018 to propose a method for determining the short-term deflection of RC beams combined with Glass Fiber Reinforced Polymer (GFRP) reinforcement. In one hand, these studies mainly focused on investigating related parameters in the calculation of deflection, such as the variation of reinforcement steel section, the use of bilinear or trilinear stress-strain curves or the addition of other materials. However, these studies have not assessed the influence of simplifications on material models, as well as overlooking the interaction between these materials in the other hand.

The complex behaviors of concrete materials can be addressed using the Concrete Damage Plasticity (CDP) model, which has been developed and widely applied in recent studies [5] - [9]. In the CDP model, the elastoplastic behaviors of concrete could be defined at different stage of loads, allowing for an accurate description of the complex stress-strain curve of concrete material. This material model could be integrated into finite element (FE) analysis software, allowing for the simulation of complex material behaviors and the interaction between elements. By comparing with data from experiments, several recent studies [6], [7], [10], [11] have pointed out that the application of CDP model in RC concrete modeling could bring results closely resemble to the actual behavior of structures.

This study focuses on evaluating the short-term deflection of RC beams according to TCVN 5574:2018 by comparing them with simulated results obtained from Abaqus software. The CDP model of concrete is defined and integrated in the ABAQUS software to account for the complexity of materials. Based on the comparison with ABAQUS, necessary recommendations are proposed to enhance safety in calculating deflections according to TCVN 5574:2018. Methods for deflection calculation and material modeling will be presented in Section 2. Section 3 focuses on the discussion of results, followed by conclusions in Section 4.

2. Deflection of RC components and modeling of materials

2.1. Deflection of flexural RC components according to Vietnamese Standard TCVN 5574:2018

TCVN 5574:2018 [1] allows determining the deflection of a RC beam subjected to a bending load following the below formula:

$$f_m = \int_0^L \overline{M}_x \left(\frac{1}{r}\right)_x dx \tag{1}$$

Where:

- \overline{M}_x : is the bending moment at the section x caused by the unit load,

- and $\left(\frac{1}{r}\right)_x = \frac{M_i}{D}$ is the overall curvature at section x considering the contribution of reinforcement steel in the concrete section with or without crack.
- M_i : is the corresponding moment for different loading cases.
- $D = E_{b1}I_{red}$: is the equivalent stiffness of the section depending on the duration of load and the presence of cracks.
- E_{b1} : is the strain modulus of the concrete under compression, depending on the duration of load and the presence of cracks.
- I_{red}: is the moment of inertia of equivalent cross-section including the section of concrete and reinforcing bars.

The values of E_{b1} and I_{red} are determined separately for cracked and non-cracked sections as presented in Eq. (2) and (3):

$$\begin{cases} E_{b1} = 0.85 \frac{E_b}{1 + \varphi_{b,cr}} & \text{for non - cracked section} \\ E_{b1} = E_{b,red} = \frac{R_{b,ser}}{\varepsilon_{b1,red}} & \text{for cracked section} \\ \begin{cases} I_{red} = I_b + \alpha(I_s + I_s') & \text{for non - cracked section} \\ I_{red} = I_b + I_s \alpha_{s2} + I_s' \alpha_{s1} & \text{for cracked section} \end{cases}$$
(3)

Where:

- E_b is the modulus of elasticity of concrete,
- $\varphi_{b,cr}$ represents the creep coefficient of the concrete ($\varphi_{b,cr} = 0$ for short-term loads), I_b, I_s, I_s' is the moment of inertia of the concrete, tension and compression reinforcement,
- $\alpha = \frac{E_s}{E_h}$ is a steel to concrete coefficient to convert steel's section into equivalent section
- $\alpha_{s1} = \frac{E_s}{E_{b,red}}$ and $\alpha_{s2} = \frac{E_s}{\psi_s E_{b,red}}$ are steel to concrete coefficient for tension and compression reinforcement respectively,
- $\varepsilon_{b1,red}$ is the relative strain of concrete in tension,
- $\psi_s = 1 0.8 \frac{M_{crc}}{M}$ is the coefficient that accounts for the uneven distribution of relative deformations of the reinforcement between cracks.

In the above equations, cracks occur when the moment generated by external forces M exceeds the cracking moment of the section (Eq. (4)).

$$M > M_{crc} = R_{h,ser} W_{nl} \tag{4}$$

Where:

- $R_{b,ser}$ is the nominal compressive strength of concrete,
- W_{pl} is the elasto-plastic flexural resistance moment of the section.

The mid-span deflection will be calculated by discretizing formula (1) in n elements (Eq.(5)), which involves dividing the component into multiple segments. The curvature at the boundaries of these segments will be determined and then multiplied by the corresponding bending moment diagram.

$$f_{m} = \frac{L^{2}}{12n^{2}} \left\{ \left(\frac{1}{r}\right)_{sup,L} + \left(\frac{1}{r}\right)_{sup,r} + 6\sum_{i=1}^{\frac{n}{2}-1} i\left[\left(\frac{1}{r}\right)_{i,L} + \left(\frac{1}{r}\right)_{i,r}\right] + (3n-2)\left(\frac{1}{r}\right)_{c} \right\}$$
(5)

The TCVN 5574:2018 will be employed to determine the deflection of a simple RC beam where its geometries, material properties are provided in Section 2.3.

2.2. Concrete Damage Plasticity (CDP) modeling

Concrete used in construction is frequently recognized as an elastoplastic material, wherein the elastic properties are considered under small loads and the plastic properties are significant during important load stages. Considering the plastic behaviors of concrete is therefore very important to have an accurate evaluation of the performance of RC structures. Toward this aim, the Concrete Damage Plasticity (CDP) is introduced in numerical modeling, allowing for an approximate description of the non-elastic behaviors and damages of concrete in both compression and tension.

2.2.1. CDP of concrete in compression

To simulate the CDP of concrete, the stress-train relationships $(\sigma_c - \varepsilon_c)$ of concrete in compression should be provided. This information could be obtained from concrete tests or approximately computed from proposed equations in previous studies ([7], [12]). Due to the lack of experimental data, this study adopted the formulations proposed in the Eurocode 2 ([13]) to construct the $(\sigma_c - \varepsilon_c)$ curve of concrete in compression. Once this relationship is completely defined, it is possible to establish the CDP model, which was originally proposed by [5] and has recently been used in several studies ([6]-[10], [12]). In the Eurocode 2 ([13]), the curve (σ_c – ε_c) of concrete in compression (Figure 1) could be approximately determined using equations (6)-(8):

$$\sigma_c = \left(\frac{k\eta - \eta^2}{1 + (k - 2)\eta}\right) f_{cm} \tag{6}$$

Where:

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \tag{7}$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \tag{7}$$

$$k = 0.15E_{cm} \frac{\varepsilon_{c1}}{f_{cm}}$$

In the equations (6)-(8), E_{cm} is the elastic modulus (GPa) of concrete; f_{cm} is the ultimate compressive strength of concrete with the corresponding peak strain ε_{c1} and ε_{cu1} is the ultimate strain ($\varepsilon_{cu1} = 0.0035$ [13]). According to Figure 1, when the applied loads are relatively small corresponding to $\sigma_c \leq \sigma_{c0} = 0.4 f_{cm}$, the relationship $\sigma_c - \varepsilon_c$ is considered linear (following the Hooke's law).

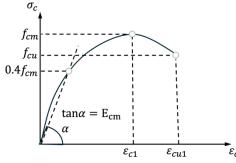


Figure 1. The stress-strain curve of concrete in compression proposed by Eurocode 2

The inelastic strains (ε_c^{in}) corresponding to each level of compression stress in concrete are determined by subtracting the elastic strains (ε_{c0}^{el}) of undamaged material from the total strain (ε_c) :

$$\varepsilon_c^{in} = \varepsilon_c - \varepsilon_{c0}^{el} \tag{9}$$

$$\varepsilon_c^{in} = \varepsilon_c - \varepsilon_{c0}^{el}$$

$$\varepsilon_{c0}^{el} = \frac{\sigma_{c0}}{E_{cm}}$$

$$(10)$$

The degradation of elastic stiffness which is represented by the compressive damage parameter d_c (or tensile damage parameter d_t for concrete in tension) could be defined for each level of inelastic strain (Figure 2) using Eq. (11).

$$d_{c} = 0 \text{, for } \varepsilon_{c} < \varepsilon_{c1}$$

$$d_{c} = \frac{f_{cm} - \sigma_{c}}{f_{cm}} \text{, for } \varepsilon_{c} \ge \varepsilon_{c1}$$

$$\sigma_{c0} = 0.4f_{cm}$$

$$(1 - d_{c})E_{cm}$$

$$\varepsilon_{c}^{in} + \varepsilon_{c0}^{el}$$

Figure 2. Concrete Damage Plasticity in Compression

The plastic strain is then determined using Eq. (12) which is always positive:
$$\varepsilon_c^{pl} = \varepsilon_c^{in} - \frac{d_c}{(1 - d_c)} \frac{\sigma_{c0}}{E_{cm}} \tag{12}$$

2.2.2. CDP of concrete in tension

For concrete in tension, various models could be found in the literature ([7], [9], [11], [14]) to describe the relationship between stress and strain. The main difference between these models mainly relates to the complexity in the approximation of $\sigma_t - \varepsilon_t$ curve during the tension stiffening stage of concrete. In this study, a linear model is adopted to describe the relationship $\sigma_t - \varepsilon_t$ of concrete at this stage (Figure 3). Assuming that the tensile strength of concrete is taken as 10% ([9], [11]) of its ultimate compressive strength ($f_t = 0.1 f_{cm}$) and the tensile strain at failure (ε_0) is equal to 10 times of tensile strain at cracking point (ε_{cr}) .

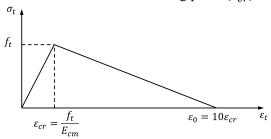


Figure 3. The stress-strain curve of concrete in tension

The cracking strain of concrete (ε_t^{ck}) is obtained by removing the elastic part (ε_{0t}^{el}) from total tensile strain (ε_t) and the tensile damage parameter (d_t) is calculated similarly to the case of compression as Eq. (13)-(15):

$$\varepsilon_t^{ck} = \varepsilon_t - \varepsilon_{0t}^{el} \tag{13}$$

$$\varepsilon_t^{ck} = \varepsilon_t - \varepsilon_{0t}^{el}$$

$$\varepsilon_{0t}^{el} = \frac{\sigma_t}{E_{cm}}$$
(13)

$$\begin{aligned} d_t &= 0 \text{ , for } \varepsilon_t < \varepsilon_{cr} \\ d_t &= \frac{f_t - \sigma_t}{f_t} \text{ , for } \varepsilon_t \ge \varepsilon_{cr} \end{aligned} \tag{15}$$

The plastic strain of concrete in tension is calculated using Eq. (16):

$$\varepsilon_t^{pl} = \varepsilon_t^{ck} - \frac{d_t}{(1 - d_t)} \frac{\sigma_t}{E_{cm}} \tag{16}$$

2.3. Numerical modeling of RC beam using ABAQUS

2.3.1. Description of investigated beam

This section focuses on studying the deflection of a simple RC beam which is supported by two locations with a distance of 2 m (Figure 4). This beam has a rectangular cross-section with a width of 150 mm and a depth of 200 mm. The thickness of the concrete cover is selected at 20 mm for all faces of the beam. The reinforcing bars for the top and bottom layers are $2\phi12$ and $3\phi12$ respectively and the shear bars $(\phi6)$ are uniformly distributed along the beam with a distance of 100 mm.

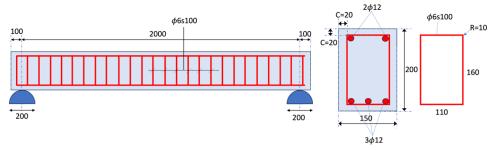


Figure 4. Details of the investigated beam

Material properties of the investigated beam are presented in Table 1 and this information are further employed to determine the CDP parameters which are previously presented in Section 2.2. Reinforcing bars and shear bars use the same steel material with the assumption that the stress-strain relationship follows a bilinear model. For safety reasons, most of RC structures are not designed to work beyond the yielding point of reinforcing bars. Therefore, while this assumption does not reflect the real stress-strain relationship of steel, it could be sufficient to describe the real behaviors of RC structure before yielding point of steel.

Material	Parameter	Description	Value
Concrete	$E_{cm} (\times 10^3 \mathrm{MPa})$	Young's modulus of concrete	31
	$ u_c$	Poisson's ratio of concrete	0.2
	f_{cm} (MPa)	Ultimate strength of concrete	33
Steel	$E_s (\times 10^3 \mathrm{MPa})$	Young's modulus of steel	200
	$ u_{\scriptscriptstyle S}$	Poisson's ratio of steel	0.3
	f_y (MPa)	Yield strength of steel	300
	f_u (MPa)	Ultimate strength of steel	440

Table 1. Material properties of the studied beam

2.3.2. Modeling of RC beam in ABAQUS

In this section, the RC beam detailed in section 2.3.1 will be modeled using a finite element software ABAQUS (Figure 5). The meshing size is 20 mm for both concrete and steel components, ensuring a good level of convergence of results as proposed in previous studies [7], [12]. Concrete elements are modelled using element type C3D8R meanwhile elements type T3D32 are selected for both reinforcing and shear bars. It is recommended to use the bond-slip model to represent the interaction between steel and concrete, especially in cases where there is a reduction in bond strength [7], [8] (e.g. for RC structures subjected to corrosion). In this study, assuming that there is

no deterioration on the bond strength between steel and concrete, and the steel elements are considered to be embedded in concrete elements. However, further developments of this study should integrate the bond-slip model to improve the accuracy of results.

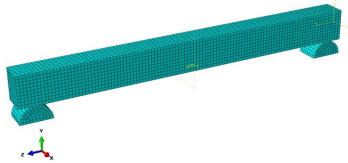


Figure 5. Modeling RC beam in ABAQUS

3. Results and discussion

Figure 6 presents the results on the mid-span deflections of the studied beam from ABAQUS and the TCVN 5574:2018 standard. It can be seen that TCVN 5574:2018 standard does not give an accurate prediction of deflection when reinforcing bars begin to yield at a loading level of 30.75 N/mm. The differences between the two approaches are more significant from the yielding point of steel onwards. This behavior is well recognized in the TCVN 5574:2018, where the deflection of an RC beam is determined with the assumption that steel is an elastic material. Consequently, the performance of the beam beyond the yielding point of reinforcing bars would not be addressed accurately. On the other hand, the modelling of steel in ABAQUS allows this material to continue working beyond the yielding point until it reaches its ultimate strength. Due to large deformations of reinforcing bars, cracks in concrete develop significantly in this stage (Figure 7c). However, at lower levels of loads before the yielding point, there is good agreement between these two approaches when the slopes of deflection curves change significantly at the point of initial cracking (Figure 6). For both methods, this cracking point corresponds to an applied load of 10.5 N/mm (Figure 7a).

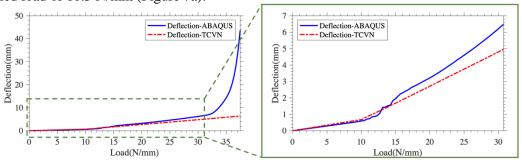


Figure 6. Comparison of deflection computed from ABAQUS and TCVN 5574:2018

For small loads (<10.5 N/mm) corresponding to the uncracking stage (elastic stage), the prediction of deflection according to TCVN5574:2018 is higher than those calculated from ABAQUS (Figure 6). This trend could be explained by the fact that while the TCVN 5574:2018 could consider the presence of reinforcing bars in deflection calculation, it cannot account for the interaction between steel and concrete as done in ABAQUS. At higher levels of loads ranging between 10.5 to 30.75 N/mm, the plastic strains in concrete increase as cracks develop (Figure 7b). However, the TCVN 5574:2018 could not account for this behavior leading to lower estimations of deflection as compared to the ABAQUS.

http://jst.tnu.edu.vn 184 Email: jst@tnu.edu.vn

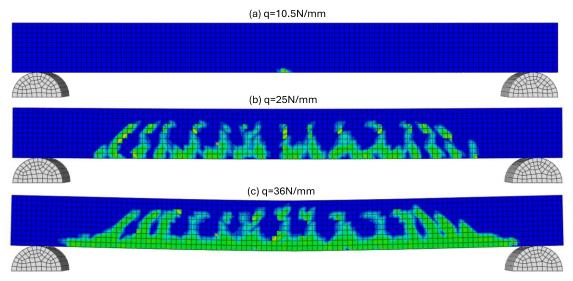


Figure 7. Tensile damage in concrete at different levels of load

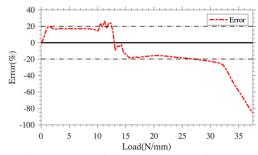


Figure 8. Relative error of deflection computed from TCVN 5574:2018

Figure 8 presents the relative errors of deflection calculated from the TCVN 5574:2018 with respect to those from ABAQUS. It is worth highlighting that these errors vary mainly in the range of (15÷20)% for applied loads corresponding to the stage of RC beam before the yielding point. This finding could be considered as a recommendation to account for these differences when using TCVN 5574:2018 to predict the deflection of RC structures.

4. Conclusions

The TCVN 5574:2018 uses simplified material models for concrete and steel in the form of bilinear or trilinear stress-strain curves. However, the behaviors of concrete and steel during the plastic stage as well as the interaction between these two materials are not well integrated leading to inaccurate estimation of deflection. When utilizing more realistic material models in a finite element model using ABAQUS, inconsistencies are found between these two approaches. Deflections calculated from TCVN 5574:2018 could be underestimated for small loads corresponding to the non-cracking stage of concrete. They may also be overestimated for higher levels of loads before reaching the yielding point of steel. Based on the results of this study, it is recommended that, for a better estimation of deflection, the TCVN 5574:2018 should take into account a difference of (15÷20)%. Future work on this study could involve considering: the modelling of cracks for the comparison of strain between steel and concrete, the long-term deflection by introducing the creep model and developing an experimental test to compare with real data.

REFERENCES

- [1] Vietnam Institute for Building Science and Technology (IBST), TCVN 5574:2018, National Standard Design of Concrete and Reinforced Concrete Structures, 2018.
- [2] N. L. Nguyen, N. T. Nguyen, V. Q. Nguyen, and Q. M. Phan, "Theoretical and experimental study for determining ultimate flexural moment of reinforced concrete beams using non-linear model of concrete deformation," *Journal of Science and Technology in Civil Engineering*, vol. 62, no. 1, pp. 36–41, 2020.
- [3] H. A. T. Nguyen, "Investigation of Bending Behaviors in Reinforced Concrete Beam from Moment-Curivature Diagrams According to TCVN 5574:2018," *Journal of Science and Technology Construction*, vol. 2, pp. 62–69, 2020.
- [4] M. T. Phan and V. T. Tran, "Research on Calculation Short-Term Deflection of Reinforced Concrete Beam Using Hybrid (Steel and GFRP) Bars Conforming to TCVN 5574:2018," *Journal of Science and Technology in Civil Engineering*, vol. 16, no. 3V, pp. 74–85, 2022.
- [5] J. Lubliner, J. Oliver, S. Oller, and E. Onate, "A Plastic-Damage Model," *Int. J. Solids Struct.*, vol. 25, no. 3, pp. 299–326, 1989.
- [6] M. H. Le, S. Khatir, M. Abdel Wahab, and T. C. Le, "A concrete damage plasticity model for predicting the effects of compressive high-strength concrete under static and dynamic loads," *Journal of Building Engineering*, vol. 44, July 2021, Art. no. 103239, doi: 10.1016/j.jobe.2021.103239.
- [7] C. V. Nguyen, Q. H. Bui, and P. Lambert, "Experimental and numerical evaluation of the structural performance of corroded reinforced concrete beams under different corrosion schemes," *Structures*, vol. 45, pp. 2318–2331, April 2022, doi: 10.1016/j.istruc.2022.10.043.
- [8] L. Amleh and A. Ghosh, "Modeling the effect of corrosion on bond strength at the steel-concrete interface with finite-element analysis," *Canadian Journal of Civil Engineering*, vol. 33, no. 6, pp. 673–682, 2006, doi: 10.1139/L06-052.
- [9] M. Hafezolghorani, F. Hejazi, R. Vaghei, M. S. B. Jaafar, and K. Karimzade, "Simplified damage plasticity model for concrete," *Structural Engineering International*, vol. 27, no. 1, pp. 68–78, 2017, doi: 10.2749/101686616X1081.
- [10] B. Alfarah, F. López-Almansa, and S. Oller, "New methodology for calculating damage variables evolution in Plastic Damage Model for RC structures," *Eng. Struct.*, vol. 132, pp. 70–86, 2017, doi: 10.1016/j.engstruct.2016.11.022.
- [11] R. Shamass, X. Zhou, and G. Alfano, "Finite-Element Analysis of Shear-Off Failure of Keyed Dry Joints in Precast Concrete Segmental Bridges," *Journal of Bridge Engineering*, vol. 20, no. 6, 2015, doi: 10.1061/(asce)be.1943-5592.0000669.
- [12] A. Mathern and J. Yang, "A practical finite element modeling strategy to capture cracking and crushing behavior of reinforced concrete structures," *Materials*, vol. 14, no. 3, pp. 1–26, 2021, doi: 10.3390/ma14030506.
- [13] British Standard Institution (BSI), "Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings," BS EN 1992-1-1:2004, London, 2004.
- [14] A. Nafees, M. F. Javed, M. A. Musarat, M. Ali, F. Aslam, and N. I. Vatin, "FE Modelling and Analysis of Beam Column Joint Using Reactive Powder Concrete," *Crystals (Basel)*, vol. 11, no. 11, 2021, doi: 10.3390/cryst11111372.