FORMATION STABILIZATION OF MOBILE AGENTS USING LOCAL POTENTIAL FUNCTIONS

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ABSTRACT
We present a constructive method to design cooperative controllers that force a group of N mobile agents to stabilize at a desired location in terms of both shape and orientation while guaranteeing no collisions between the agents. The control development is based on new local potential functions, which attain the minimum value when the desired formation is achieved, and are equal to infinity when a collision occurs. Several simulation examples are included to illustrate the approach throughout the paper.
Keywords: *Formation stabilization, mobile agents, local potential functions, ocean vehicles*

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TÓM TẮT
Trình bày một phương pháp hệ thống để thiết kế các bộ điều khiển ổn định phối hợp cho một nhóm N thiết bị di động tại một vị trí định trước cả về hình dạng và hướng, và đảm bảo không có va chạm giữa các thiết bị. Các bộ điều khiển được thiết kế dựa trên các hàm thế năng nhân tạo mới có giá trị cực thiểu khi các thiết bị di động được ổn định tại vị trí định trước và đạt giá trị vô hạn khi xảy ra va chạm. Bài báo cũng bao gồm một số ví dụ minh họa.
Từ khóa: *Ổn định nhóm, thiết bị di động, phương tiện giao thông đường biển*

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INTRODUCTION

Formation control of multiple mobile agents has received a lot of attention from the control community over the last few years. Applications of vehicle formation control include the coordination of multiple robots, unmanned air/ocean vehicles, satellites, aircraft and spacecraft 0-[32]. For example, a group of mobile vehicles can be used to carry out tasks that are difficult or not effective for a single vehicle to perform alone. In the literature, there are roughly three methods to formation control of multiple vehicles: leader-following, behavioral and virtual structure. Each method has its own advantages and disadvantages. In the leader-following approach, some vehicles are considered as leaders, whist the rest of robots in the group act as followers 0, 0, 0, 0. The leaders track predefined reference trajectories, and the followers track transformed versions of the states of their nearest neighbors according to given schemes. An advantage of the leader-following approach is that it is easy to understand and implement. In addition, the formation can still be maintained even if the leader is perturbed by some disturbances. However, a disadvantage is that there is no explicit feedback to the formation, that is, no explicit feedback from the followers to the leader in this case. If the follower is perturbed, the formation cannot be maintained. Furthermore, the leader is a single point of failure for the formation. In the behavioral approach 0, 0, 0, 0, 0, 0, 0, few desired behaviors such as collision/obstacle avoidance and goal/target seeking are prescribed for each vehicle and the formation control is calculated from a weighting of the relative importance of each behavior. The advantages of this approach are: it is natural to derive control strategies when vehicles have multiple competing objectives, and an explicit feedback is included through communication between neighbors. The disadvantages are: the group behavior cannot be explicitly defined, and it is difficult to analyze the approach mathematically and guarantee the group stability. In the virtual structure approach, the entire formation is treated as a single entity 0, 0, 0, 0. When the structure moves, it traces out desired trajectories for each agent in the group to track. Some similar ideas based on the perceptive reference frame, the virtual leader, and the formation reference point are given in 0, 0, 0 respectively. The advantages of the virtual structure approach are: it is fairly easy to prescribe the coordinated behavior for the group, and the formation can be maintained very well during the manoeuvres, i.e. the virtual structure can evolve as a whole in a given direction with some given orientation and maintain a rigid geometric relationship among multiple vehicles. However requiring the formation to act as a virtual structure limits the class of potential applications such as when the formation shape is time-varying or needs to be frequently reconfigured, this approach may not be the optimal choice. The virtual structure and leader-following approaches require that the full state of the leader or virtual structure be communicated to each member of the formation. In contrast, behavior-based approach is decentralized and may be implemented with significantly less communication. Formation feedback has been recently introduced in the literature 0, 0, 0, 0. In 0, a coordination architecture for spacecraft formation control is introduced to incorporate the leader-following, behavioral, and virtual structure approaches to the multi-agent coordination problem. This architecture can be extended to include formation feedback. In 0, formation feedback is used for the coordinated control problem for multiple robots. In 0, a Lyapunov formation function is used to define a formation error for a class of robots (double integrator dynamics) so that a constrained motion control problem of multiple systems is converted into a stabilization problem for one single system. The error feedback is incorporated to the virtual leader through parameterized trajectories.

The formation control problem for the three general approaches described above would be for each agent to move to a desired point in the formation while avoiding collisions. Such a desired point may be time varying or stationary, and can be defined, for instance, relative to a leader or virtual structure. The objective can be achieved through the use of centralized control, see for example 0, by using a single controller that generates collision free trajectories in the workspace. Although this guarantees a complete solution, centralized schemes require high computational power (on the part of the central command and control centre) and are not robust due to the heavy dependence on a single controller. On the other hand, decentralized schemes, see for example 0, require less computational effort, and is relatively more scalable to team size. This approach usually involves a combination of agent based local potential fields 0, **Error! Reference source not found.**. The main problem with the decentralized approach is that it is unable or extremely difficult to predict and control the critical points, i.e. the closed loop system has multiple equilibrium points. It is rather difficult to design a controller such that all the equilibrium points except for the desired equilibrium ones (in the formation that the agents are to track) are unstable/saddle points. Recently, following the approach presented in 0, a method based on a different navigation function provided a centralized formation stabilization control design strategy, which can potentially be extended for complete decentralization, is proposed in 0. However, the navigation function approaches a finite value when a collision occurs, and the formation is stabilized to any point in workspace instead of being “tied” to a fixed coordinate frame. This motivates our work presented in this paper, which derives control laws for the agents to track their desired locations within formations, and such that only the critical points at the desired locations in the formation are stable.

In this paper, a constructive method is proposed to design cooperative controllers to solve the problem of stabilizing a group of N mobile agents at a (pre-specified) desired location in terms of both shape and orientation while avoiding collisions between themselves. The control development is based on new local potential functions guaranteeing global and complete convergence except for the set of measure zero. These local potential functions are chosen such that when the controls are designed to decrease these functions, all the agents approach their desired locations and no collisions can occur. Behavior of the closed loop system near equilibrium points is investigated via linearization of the inter-agent dynamics around those points. We also show that the proposed control scheme is easy to extend to design bounded controllers.

The rest of the paper is organized as follows. In the next section, we present a simple example in two-dimensional space to illustrate the approach. Section 3 presents the control design and stability analysis for formation stabilization. Section 4 concludes our paper.

PLANAR FORMATION STABILIZATION OF TWO AGENTS

To illustrate our proposed approach to solve the problem of formation stabilization and formation tracking of N mobile agents, we begin with an examination of a group of two mobile agents whose dynamics are given by

$$\dot{q}_i = u_i \quad (1)$$

where $q_i = [x_i\ y_i]^T \in \mathbb{R}^2$ and $u_i = [u_{ix}\ u_{iy}]^T \in \mathbb{R}^2$, $i = 1,2$ are the states and control inputs of agents 1 and 2, respectively. The control objective is to design the controls u_i such that they force the agents to move from initial positions $q_i(t_0)$, $t_0 \geq 0$ to final positions $q_{if} = [x_{if}\ \ y_{if}]^T$ while avoiding collisions between the agents. It is indeed assumed that the initial and the final positions of the agent 1 are different from those of the agent 2, i.e. $\| q_1(t_0) - q_2(t_0) \| > 0$ and $\| q_{1f} - q_{2f} \| > 0$, where $\| \bullet \|$ denotes the standard Euclidian norm of $\bullet$.

Control design

Consider the following potential function

$$\varphi_i = \gamma_i + \delta\beta_i \tag{2}$$

where δ is a positive tuning constant, γ_i and β_i are the goal and related collision avoidance functions, respectively. They are specified below.

-The goal function γ_i is designed such that it puts penalty on the stabilization error for the agent i, and is equal to zero when the agent is at its final position. A simple choice of this function is

$$\gamma_i = 0.5 \| q_i - q_{if} \|^2 . \tag{3}$$

-The related collision avoidance function β_i is designed such that it is equal to infinity a collision occurs, and attains the minimum value when the agents move in the desired formation. A possible choice of this function is

$$\beta_i = \frac{\beta_{ij}}{\beta_{ijf}^2} + \frac{1}{\beta_{ij}},\ (i,j) \in (1,2),\ i \neq j \tag{4}$$

where

$$\beta_{ij} = 0.5 \| q_i - q_j \|^2,\ \beta_{ijf} = 0.5 \| q_{if} - q_{jf} \|^2 . \tag{5}$$

To design the controls $u_i = [u_{ix}\ u_{iy}]^T$, differentiating both sides of (2) along the solutions of (1) gives

$$\dot{\varphi}_i = \Omega_{ix}u_{ix} + \Omega_{iy}u_{iy} - \delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(x_i - x_j)u_{jx} - \delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(y_i - y_j)u_{jy} \tag{6}$$

where

$$\begin{aligned} \Omega_{ix} &= \left[x_i - x_{if}\right] + \left[\delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(x_i - x_j)\right] \\ \Omega_{iy} &= \left[y_i - y_{if}\right] + \left[\delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(y_i - y_j)\right]. \end{aligned} \tag{7}$$

The equation (6) suggests that we choose the controls $u_i = [u_{ix}\ u_{iy}]^T$ as

$$\begin{cases} u_{ix} = -c\Omega_{ix} \\ u_{iy} = -c\Omega_{iy} \end{cases} \tag{8}$$

where c is a positive constant. Substituting (8) into (6) yields

$$\dot{\varphi}_i = -c(\Omega_{ix}^2 + \Omega_{iy}^2) - \delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(x_i - x_j)u_{jx} - \delta\left(1/\beta_{ijf}^2 - 1/\beta_{ij}^2\right)(y_i - y_j)u_{jy} . \tag{9}$$

Indeed, substituting (8) into (1) results in the closed loop system

$$\dot{q}_i = -c\Omega_i,\ i = 1,2 \ (10)$$

where $\Omega_i = [\Omega_{ix}\ \Omega_{iy}]^T$.

Remark 1. The control pairs (u_{1x}, u_{2x}) and (u_{1y}, u_{2y}) have a special feature in the sense that the first terms (see first square brackets in Ω_{ix} and Ω_{iy} in (7)) play the role of driving the agents to their final positions while the second terms (see second square brackets in Ω_{ix} and Ω_{iy} in (7)) act as both attractive and repulsive forces to attract the agents when the distance between them is larger than the desired distance, i.e. when

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} > \sqrt{(x_{1f} - x_{2f})^2 + (y_{1f} - y_{2f})^2} \tag{11}$$

and push the agents away from each other when the distance smaller than the desired one, i.e. when

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}<\sqrt{(x_{1f}-x_{2f})^2+(y_{1f}-y_{2f})^2}\ . \tag{12}$$

The second terms act as gyroscopic forces 0 to steer the agents away from each other when they come to close to each other.

Stability analysis

In this subsection, we show that the controls $u_i=[u_{ix}\ u_{iy}]^T$ given in (8) guarantees no collisions occur, the solutions of the closed loop system (10) exist, and the agents move to their desired positions asymptotically.

-Proof of no collisions and existence of solutions.

Consider the following global potential function

$$\varphi=\sum_{i=1}^{2}(\varphi_i-0.5\delta\beta_i)\ . \tag{13}$$

The function φ is a proper function since substituting (2) and (4) into (13) results in

$$\varphi=\gamma_1+\gamma_2+\delta\left(\frac{\beta_{12}}{\beta_{12f}^2}+\frac{1}{\beta_{12}}\right) \tag{14}$$

which is positive definite, radially unbounded with respect to the stabilization errors $\|q_1-q_{1f}\|$ and $\|q_2-q_{2f}\|$, and is equal to infinity when a collision between the agent 1 and agent 2 occurs. Differentiating both sides of (14) along the solutions of the closed loop system (10) results in

$$\dot{\varphi}=-c\sum_{i=1}^{2}\Omega_i^T\Omega_i\ . \tag{15}$$

From (15), we have $\dot{\varphi}\le 0$. Integrating both sides of this inequality gives

$$\sum_{i=1}^{2}\gamma_i(t)+\delta\left(\frac{\beta_{12}(t)}{\beta_{12f}^2}+\frac{1}{\beta_{12}(t)}\right)\le\sum_{i=1}^{2}\gamma_i(t_0)+\delta\left(\frac{\beta_{12}(t_0)}{\beta_{12f}^2}+\frac{1}{\beta_{12}(t_0)}\right),\ \forall t\ge t_0\ge 0 \tag{16}$$

where

$$\begin{aligned}&\gamma_i(t)=0.5\|q_i(t)-q_{if}\|^2,\ \gamma_i(t_0)=0.5\|q_i(t_0)-q_{if}(t_0)\|^2,\ i=1,2\\&\beta_{12}(t)=0.5\|q_1(t)-q_2(t)\|^2,\ \beta_{12}(t_0)=0.5\|q_1(t_0)-q_2(t_0)\|^2\end{aligned} \tag{17}$$

Since $\|q_1(t_0)-q_2(t_0)\|>0$ and $\|q_{1f}-q_{2f}\|>0$, i.e. $\beta_{12}(t_0)>0$ and $\beta_{12f}>0$, the right hand side of (16) is bounded. As a result, the left hand side of (16) must also be bounded. This means that $\beta_{12}(t)>0$, $\forall t\ge t_0\ge 0$, i.e. no collisions between the agents can occur. Boundedness of the left hand side of (16) also implies that of $\|q_1(t)\|$ and $\|q_2(t)\|$, i.e. the solutions of the closed loop system (10) exist. Furthermore, applying Barbalat's lemma found in [ ] to (15) gives

$$\lim_{t\to\infty}\Omega_i(t)=0,\ i=1,2\ . \tag{18}$$

-Behavior near equilibrium points.

At the steady state, we have $\Omega_1=0$ and $\Omega_2=0$. These equations have two set of roots $q_1=q_{1f},\ q_2=q_{2f}$ and $q_1=q_{1c},\ q_2=q_{2c}$. Since the obstacle function is specified in terms of relative distance between the agents, it is easier to investigate behavior of the closed loop system near the equilibrium points by considering the inter-agent dynamics instead of dynamics of each individual agent. Defining $q_{12}=q_1-q_2$ and differentiating this equation along the solutions of the closed loop system (10) yield

$$\dot{q}_{12}=-c\Omega_{12} \tag{19}$$

where $\Omega_{12}=\Omega_1-\Omega_2$. We can write Ω_{12} as a vector function of q_{12} and $q_{12f}=q_{1f}-q_{2f}$ as $\Omega_{12}=q_{12}-q_{12f}+2\delta\left(1/\beta_{12f}^2-1/\beta_{12}^2\right)q_{12}$. At the steady state, we have $\Omega_{12}=0$ since $\Omega_1=0$ and $\Omega_2=0$. The equation $\Omega_{12}=0$ has two roots $q_{12}=q_{12f}, q_{12f}=q_{1f}-q_{2f}$ and $q_{12}=q_{12c}, q_{12c}=q_{1c}-q_{2c}$. Therefore (18) implies that q_{12} approaches either q_{12f} or q_{12c}. Since substituting $q_{12}=q_{12f}$ or $q_{12}=q_{12c}$ into the equations $\Omega_1=0$ and $\Omega_2=0$ results in $q_1=q_{1f}, q_2=q_{2f}$ and $q_1=q_{1c}, q_2=q_{2c}$, we just need to investigate behavior of the system (19) near the equilibrium points q_{12f} and q_{12c}. Before going further, it is noted that q_{12c} has a property that the term $q_{12c}^T q_{12f}$ is strictly negative, i.e. the point at which $q_{12}=[0\ 0]^T$ locates between the equilibrium point q_{12c} and the equilibrium point q_{12f}. This is because of the fact that at the equilibrium point q_{12f} all the forces (attractive and repulsive) are equal to zero while at the critical point q_{12c} the sum of attractive and repulsive forces (but they are different from zero) is equal to zero. This can be viewed graphically in Figure 1.

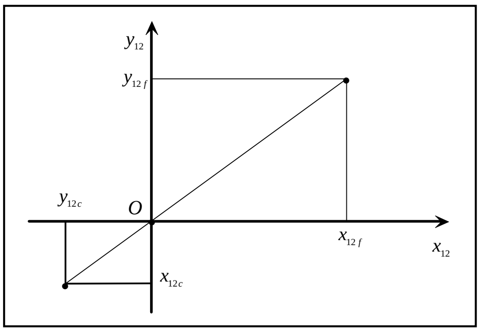

Figure 1. *Illustrating location of equilibrium points*

We will show that the equilibrium point q_{12f} is asymptotically stable while the equilibrium point q_{12c} is saddle. The general gradient of $\Omega_{12}(q_{12},q_{12f})$ with respect to q_{12} is given by

$$\frac{\partial\Omega_{12}}{\partial q_{12}}=\begin{bmatrix}1+2\delta\left(1/\beta_{12f}^2-1/\beta_{12}^2\right)+4\delta x_{12}^2/\beta_{12}^3 & 4\delta x_{12}y_{12}/\beta_{12}^3\\ 4\delta x_{12}y_{12}/\beta_{12}^3 & 1+2\delta\left(1/\beta_{12f}^2-1/\beta_{12}^2\right)+4\delta y_{12}^2/\beta_{12}^3\end{bmatrix} \quad (20)$$

where x_{12} and y_{12} are defined from $q_{12}=[x_{12}\ y_{12}]^T$. To show that the equilibrium point q_{12f} is asymptotically stable, we need to show that the matrix $A_{q_{12f}}=\left.\frac{\partial\Omega_{12}}{\partial q_{12}}\right|_{q_{12}=q_{12f}}$ is positive definite.

Substituting $q_{12}=q_{12f}$ into (20) yields

$$A_{q_{12f}}=\begin{bmatrix}1+4\delta x_{12f}^2/\beta_{12f}^3 & 4\delta x_{12f}y_{12f}/\beta_{12f}^3\\ 4\delta x_{12f}y_{12f}/\beta_{12f}^3 & 1+4\delta y_{12f}^2/\beta_{12f}^3\end{bmatrix}. \quad (21)$$

Since $1+4\delta x_{12f}^2/\beta_{12f}^3>0$ and $\det(A_{q_{12f}})=1+4\delta/\beta_{12f}^2>0$ where $\det(\bullet)$ denotes the determinant of $\bullet$, the matrix $A_{q_{12f}}$ is positive definite, i.e. the equilibrium point q_{12f} is asymptotically stable.

On the other hand at the equilibrium point q_{12c}, we have

$$\left.\frac{\partial\Omega_{12}}{\partial q_{12}}\right|_{q_{12}=q_{12c}}=\begin{bmatrix}1+2\delta\left(1/\beta_{12f}^2-1/\beta_{12c}^2\right)+4\delta x_{12c}^2/\beta_{12c}^3 & 4\delta x_{12c}y_{12c}/\beta_{12c}^3\\ 4\delta x_{12c}y_{12c}/\beta_{12c}^3 & 1+2\delta\left(1/\beta_{12l}^2-1/\beta_{12c}^2\right)+4\delta y_{12c}^2/\beta_{12c}^3\end{bmatrix}\triangleq A_{q_{12c}}$$

(22)

where x_{12c} and y_{12c} are defined from $q_{12c} = [x_{12c}\ y_{12c}]^T$ and $\beta_{12c} = 0.5\| q_{12c} \|^2$. The determinant of the matrix $A_{q_{12c}}$ is given by

$$\det(A_{q_{12c}}) = \left(1 + 2\delta\left(1/\beta_{12f}^2 - 1/\beta_{12c}^2\right)\right)\left(1 + 2\delta\left(1/\beta_{12f}^2 + 3/\beta_{12c}^2\right)\right). \quad (23)$$

Since at the equilibrium point q_{12c}, we have $\Omega_{12c} = 0$ where Ω_{12c} is Ω_{12} being evaluated at $q_{12} = q_{12c}$. Multiplying both sides of $\Omega_{12c} = 0$ with q_{12c}^T, we have $q_{12c}^T \Omega_{12c} = 0$. Expanding $q_{12c}^T \Omega_{12c} = 0$ gives

$$2\delta\left(1/\beta_{12f}^2 - 1/\beta_{12c}^2\right) = -\frac{1}{2\beta_{12c}} q_{12c}^T (q_{12c} - q_{12f}) . \quad (24)$$

Substituting (24) into (23) yields

$$\det(A_{q_{12c}}) = \frac{q_{12c}^T q_{12f}}{2\beta_{12c}}\left(1 + 2\delta\left(1/\beta_{12f}^2 + 3/\beta_{12c}^2\right)\right) . \quad (25)$$

Since $q_{12c}^T q_{12f}$ is strictly negative, we have $\det(A_{q_{12c}}) < 0$, which implies that the equilibrium point q_{12c} is saddle. □

FORMATION STABILIZATION OF N AGENTS

In this section, we extend the results obtained for the simple system presented in the previous section to a more complex system of N mobile agents.

Problem statement

We consider a group of N mobile agents, of which each has the following dynamics

$$\dot{q}_i = u_i,\ i = 1,...,N \quad (26)$$

where $q_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ are the state and control input of the agent i. We assume that $n > 1$ and $N > 1$. In this paper, we treat each agent as an autonomous point. The assumption that each agent is represented as a point is not as restrictive as it may seem since various shapes can be mapped to single points through a series of transformations 0, 0, 0. Our task is to design the control input u_i for each agent i that forces the group of N agents to stabilize with respect to the fixed coordinate in a particular formation specified by a desired vector $q_f = [q_{1f}^T, q_{2f}^T, ..., q_{Nf}^T]^T$ while avoiding collisions between themselves. The control objective is formally stated as follows:

Control objective: Assume that at the initial time t_0 each agent starts at a different location, and that each agent has a different desired location, i.e. there exist strictly positive constants ε_1 and ε_2 such that

$$\| q_i(t_0) - q_j(t_0) \| \ge \varepsilon_1$$
$$\| q_{if} - q_{jf} \| \ge \varepsilon_2,\ \forall i, j \in \{1, 2, ...N\}. \quad (27)$$

Design the control input u_i for each agent i such that each agent (almost) globally asymptotically approaches its desired location while avoids collisions with all other agents in the group, i.e.

$$\lim_{t\to\infty}(q_i(t) - q_{if}) = 0$$
$$\| q_i(t) - q_j(t) \| \ge \varepsilon_3,\ \forall i, j \in \{1, 2, ...N\},\ \forall t \ge t_0 \ge 0 \quad (28)$$

where ε_3 is a strictly positive constant.

The fixed desired formation can be represented by a labeled directed graph 0 in the following definition.

Definition 1. The formation graph, $G = \{V, E, L\}$ is a directed labeled graph consisting of:

-a set of vertices (nodes), $V = \{v_1, \cdots, v_N\}$ indexed by the mobile agents in the group,

-a set of edges, $E = \{(v_i, v_j) \in V \times V\}$, containing ordered pairs of vertices that represent inter-agent position constraints, and

-a set of labels, $L = \{\gamma_{dij} \mid \gamma_{dij} = \| q_i - q_j - l_{ij} \|^2$, $\forall (v_i, v_j) \in E\}$, $l_{ij} = q_{if} - q_{jf} \in \mathbb{R}^n$ indexed by the edges in E.

Indeed, when the control objective is achieved, the edge labels become $\| q_i - q_j - l_{ij} \|^2 = 0$, $\forall (v_i, v_j) \in E$, i.e. the relative distance between the agents i and j is $l_{ij} = q_{if} - q_{jf}$.

Control design

The example in Section 2 motivates us to use the following local potential function

$$\varphi_i = \gamma_i + \delta\beta_i,\ i = 1,...,N \quad (29)$$

where δ are positive tuning constants, the functions γ_i and β_i are the goal and related collision avoidance functions for the agent i specified as follows:

-The goal function γ_i is designed such that it puts penalty on the stabilization error for the agent i, and is equal to zero when the agent is at its final position.

$$\gamma_i = \frac{1}{2}\| q_i - q_{jf} \|^2. \quad (30)$$

-The related collision function β_i should be chosen such that it is equal to infinity whenever any agents come in contact with the agent i, i.e. a collision occurs, and attains the minimum value when the agent i is at its desired location with respect to other group members belong to N_i, which are adjacent to the agent i. This function is chosen as follows:

$$\beta_i = \sum_{j\in N_i}\left(\frac{\beta_{ij}^k}{\beta_{ijf}^{2k}} + \frac{1}{\beta_{ij}^k}\right) \quad (31)$$

where k is a positive constant to be chosen later, β_{ij} and β_{ijf} are collision and desired collision functions chosen as

$$\beta_{ij} = \frac{1}{2}\| q_i - q_j \|^2,\ \beta_{ijf} = \frac{1}{2}\| q_{if} - q_{jf} \|^2. \quad (32)$$

It is noted from (32) that $\beta_{ij} = \beta_{ji}$ and $\beta_{ijf} = \beta_{jif}$.

Remark 2.

1. The above choice of the potential function φ_i given in (29) with its components specified in (30)-(32), has the following properties: 1) it attains the (unique) minimum value when the agent i is at its final position q_{if}, and 2) it is equal to infinity whenever any two or more agents come in contact with the agent i, i.e. when a collision occurs.

2. The potential function (29) is different from the one proposed in 0 and 0 in the sense that the ones in 0 and 0 are centralized and does not put penalty on the distance between the agent and its final position, i.e. does not include the goal function γ_i. Therefore, the controllers developed in 0 and 0 do not guarantee the formation converge to a specified configuration but to any configuration that minimize the potential function.

3. Our potential function (29) is also different from the navigation functions proposed in 0, 0 and 0 in the sense that our potential function is in the form of sum of collision avoidance functions while those navigation functions in the form of product of collision avoidance functions 0 and 0. This feature makes our potential function "more decentralized". Our potential function is equal to infinity while those in 0, 0 and 0 is equal to a finite constant when a collision occurs. Moreover, our potential function puts penalty on stabilization error between the agent and its final position, hence, guarantees the formation will be stabilized with respect to a fixed coordinate system instead of "loosing" in space as in 0, 0. However, those in 0, 0 and 0 also cover obstacle and work space boundary avoidance. We do not include these issues in our present paper for clarity. Including these issues is possible and is the subject of the future research.

4. Our potential function does not have problems like local minima and non-reachable goal as listed in **Error! Reference source not found.**.

To design the control input u_i, we differentiate both sides of (29) along the solutions of (26) to obtain

$$\dot{\varphi}_i = \Omega_i^T u_i - \Phi_{ij}^T u_j \quad (33)$$

where

$$\Phi_{ij} = \delta k\left(\frac{1}{\beta_{ijf}^{2k}} - \frac{1}{\beta_{ij}^{2k}}\right)\beta_{ij}^{k-1}(q_i - q_j)$$
$$\Omega_i = q_i - q_{if} + \sum_{j\in N_i}\Phi_{ij}. \quad (34)$$

From (33), we simply choose the control u_i for the agent i as follows:

$$u_i = -C\Omega_i \quad (35)$$

where $C \in \mathbb{R}_{+}^{n\times n}$ is a symmetric positive definite matrix. Substituting (35) into (33) yields

$$\dot{\varphi}_i = -\Omega_i^T C\Omega_i - \sum_{j\in N_i} \Phi_{ij}^T u_j \ . \tag{36}$$

Substituting (35) into (26) results in the closed loop system

$$\dot{q}_i = -C\Omega_i, \ i = 1,...,N \ . \tag{37}$$

Theorem 1. Under the assumptions stated in the control objective, see (27), the control for each agent i given in (35) with an appropriate choice of the tuning constants δ and k, solves the control objective.

Proof. We prove Theorem 1 in two steps. At the first step, we show that there are no collisions between any agents and the solutions of the closed loop system (37) exist. At the second step, we prove that the equilibrium point of the closed loop system (37), at which $q_i - q_{if} = 0$, is asymptotically stable. Finally, we show that all other equilibrium(s) of (37) are either unstable or saddle.

Step 1. Proof of no collision and existence of solutions:

We consider the following common potential function φ given by

$$\varphi = \sum_{i=1}^{N} (\varphi_i - 0.5\delta\beta_i) . \tag{38}$$

The function φ is indeed a proper (positive definite, radially unbounded and equal to infinity when a collision occurs) function since substituting the functions φ_i and β_i given in (29) and (31) into (38) results in

$$\varphi = \sum_{i=1}^{N} (\gamma_i + 0.5\delta\beta_i) = \sum_{i=1}^{N} \gamma_i + \delta \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{\beta_{ij}^k}{\beta_{ijf}^{2k}} + \frac{1}{\beta_{ij}^k} \right) \tag{39}$$

which is essentially sum of all goal functions and a combination of all possible related collision functions. Differentiating both sides of (38) along the solutions of (36) and the closed loop system (37), or (39) along the solutions of the closed loop system (37) yields

$$\dot{\varphi} = -\sum_{i=1}^{N} \Omega_i^T C\Omega_i \ . \tag{40}$$

From (40), we have $\dot{\varphi} \le 0$. Integrating both sides of $\dot{\varphi} \le 0$ results in $\varphi(t) \le \varphi(t_0)$. From definition of φ given in (39) we can write $\varphi(t) \le \varphi(t_0)$ as

$$\sum_{i=1}^{N} \gamma_i(t) + \delta \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{\beta_{ij}^k(t)}{\beta_{ijf}^{2k}} + \frac{1}{\beta_{ij}^k(t)} \right) \le \sum_{i=1}^{N} \gamma_i(t_0) + \delta \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{\beta_{ij}^k(t_0)}{\beta_{ijf}^{2k}} + \frac{1}{\beta_{ij}^k(t_0)} \right) \tag{41}$$

where

$$\begin{aligned} \gamma_i(t) &= \frac{1}{2} \| q_i(t) - q_{if} \|^2, \ \beta_{ij}(t) = \frac{1}{2} \| q_i(t) - q_j(t) \|^2, \\ \gamma_i(t_0) &= \frac{1}{2} \| q_i(t_0) - q_{if} \|^2, \ \beta_{ij}(t_0) = \frac{1}{2} \| q_i(t_0) - q_j(t_0) \|^2 . \end{aligned} \tag{42}$$

From (27) we have $\beta_{ij}(t_0)$ and β_{ijf} are strictly larger than some positive constants. Therefore the right hand side of (41) is bounded by some positive constant depending on the initial conditions. Boundedness of the right hand side of (41) implies that the left hand side of (41) must be also bounded. As a result, $\beta_{ij}(t)$ must be strictly larger than some positive constant denoted by ε_3 for all $t \ge t_0 \ge 0$. From definition of $\beta_{ij}(t)$, see (42), $\| q_i(t) - q_j(t) \|$ must be larger than some strictly positive constant denoted by ε_3, i.e. there are no collisions. Boundedness of the left hand side of (41) also implies that of $\| q_i(t) \|$ for all $t \ge t_0 \ge 0$, i.e. the solutions of the closed loop system (37) exist. Furthermore, applying Barbalat's lemma to (40) gives

$$\lim_{t\to\infty} \Omega_i(t) = 0 . \tag{43}$$

Step 2. Behavior near equilibrium points.

At the steady state, the equilibrium points are found by solving

$$\Omega_i = q_i - q_{if} + \delta k \sum_{j \in N_i} \left(\frac{1}{\beta_{ijf}^{2k}} - \frac{1}{\beta_{ij}^{2k}} \right) \beta_{ij}^{k-1}(q_i - q_j) = 0,\ i = 1,...,N\ . \tag{44}$$

It is directly verified that $q = q_f$ where q and q_f are stack vectors of q_i and q_{if}, respectively, is one root of the equation (44). In addition there is (are) another root(s) denoted by q_c of (44) different from q_f satisfying

$$\Omega_i\big|_{q=q_c} = q_{ic} - q_{if} + \delta k \sum_{j \in N_i} \left(\frac{1}{\beta_{ijl}^{2k}} - \frac{1}{\beta_{ijc}^{2k}} \right) \beta_{ijc}^{k-1}(q_{ic} - q_{jc}) = 0,\ i = 1,...,N \tag{45}$$

where $\beta_{ijc} = 0.5\| q_{ic} - q_{jc} \|^2$. Moreover, since the collision avoidance functions are specified in terms on relative distances between the agents, we write the closed loop system of the inter-agent dynamics from the closed loop system (37) as

$$\dot{q}_{ij} = -C(\Omega_i - \Omega_j),\ (i, j) \in \{1,...,N\},\ i \neq j \tag{46}$$

where $q_{ij} = q_i - q_j$. Defining $\bar{q}$ and $\bar{q}_f$ are stack vectors of q_{ij} and q_{ijf} with $q_{ijf} = q_{if} - q_{jf}$, respectively, i.e. $\bar{q} = [q_{12}^T, q_{13}^T,..., q_{ij}^T,..., q_{N-1,N}^T]^T$ and $\bar{q}_f = [q_{12f}^T, q_{13f}^T,..., q_{ijf}^T,..., q_{N-1,Nf}^T]^T$, we can write the closed loop system of the inter-agent dynamics (46) as

$$\dot{\bar{q}} = -\bar{C}F(\bar{q}, \bar{q}_f)\ . \tag{47}$$

$\bar{C} = \text{diag}(\underbrace{C,\cdots,C}_{E})$ with E the number of edges of the formation graph, and

$$F(\bar{q}, \bar{q}_f) = [\Omega_1^T - \Omega_2^T, \Omega_1^T - \Omega_3^T,..., \Omega_i^T - \Omega_j^T,..., \Omega_{N-1}^T - \Omega_N^T]^T\ . \tag{48}$$

In the followings, we will show that the equilibrium point $\bar{q} = \bar{q}_f$ is asymptotically stable, and the equilibrium point(s) $\bar{q} = \bar{q}_c$ is (are) unstable or saddle. Since (43) holds for all $i = 1,...,N$, at the steady state we have $\Omega_i - \Omega_j = 0,\ \forall (i, j) \in \{1,...,N\},\ i \neq j$. Therefore the equilibrium points $\bar{q} = \bar{q}_f$ and $\bar{q} = \bar{q}_c$ are also the equilibrium points of (47). The general gradient of $F(\bar{q}, \bar{q}_f)$ with respect to $\bar{q}$ is given by

$$\frac{\partial F(\bar{q}, \bar{q}_f)}{\partial \bar{q}} = \begin{bmatrix} \frac{\partial \Xi_{12}}{\partial q_{12}} & \frac{\partial \Xi_{12}}{\partial q_{13}} & \cdots & \cdots & \frac{\partial \Xi_{12}}{\partial q_{N-1,N}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Xi_{ij}}{\partial q_{12}} & \cdots & \frac{\partial \Xi_{ij}}{\partial q_{ij}} & \cdots & \frac{\partial \Xi_{ij}}{\partial q_{N-1,N}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Xi_{N-1,N}}{\partial q_{12}} & \cdots & \cdots & \cdots & \frac{\partial \Xi_{N-1,N}}{\partial q_{N-1,N}} \end{bmatrix},\ \Xi_{ij} = \Omega_i - \Omega_j,\ (i, j) \in \{1,...,N\},\ i \neq j\ . \tag{49}$$

It can be checked that

$$\begin{aligned} \frac{\partial \Xi_{ij}}{\partial q_{ij}} &= I_{n\times n} + 2\delta k \left(\frac{1}{\beta_{ijf}^{2k}} - \frac{1}{\beta_{ij}^{2k}} \right) \beta_{ij}^{k-1} I_{n\times n} + 2\delta k \left((k-1) \left(\frac{1}{\beta_{ijf}^{2k}} - \frac{1}{\beta_{ij}^{2k}} \right) \beta_{ij}^{k-2} + \frac{2k}{\beta_{ij}^{k+2}} \right) q_{ij} q_{ij}^T \triangleq H_{ij} \\ \frac{\partial \Xi_{ij}}{\partial q_{cd}} &= \sigma \delta k \left(\frac{1}{\beta_{cdf}^{2k}} - \frac{1}{\beta_{cd}^{2k}} \right) \beta_{cd}^{k-1} I_{n\times n} + \sigma \delta k \left((k-1) \left(\frac{1}{\beta_{cdf}^{2k}} - \frac{1}{\beta_{cd}^{2k}} \right) \beta_{cd}^{k-2} + \frac{2k}{\beta_{cd}^{k+2}} \right) q_{cd} q_{cd}^T, \end{aligned} \tag{50}$$

where $(c,d) \in \{1,...,N\}$, $(c,d) \neq (i,j), c \neq d$, and $\sigma = 1$ or $\sigma = -1$ depending on value of c, d, i and j. However, we do not need to specify the sign of σ for our next task. We now investigate properties of the equilibrium points $\bar{q} = \bar{q}_f$ and $\bar{q} = \bar{q}_c$ based on the general gradient $\partial F(\bar{q},\bar{q}_f)/\partial\bar{q}$ evaluated at those points.

Step 2.1 Proof of $\bar{q} = \bar{q}_f$ *being the asymptotic stable equilibrium point:*

At the equilibrium point $\bar{q} = \bar{q}_f$, we have

$$\left.\frac{\partial \Xi_{ij}}{\partial q_{ij}}\right|_{\bar{q}=\bar{q}_f} = I_{n\times n} + \frac{4\delta k^2}{\beta_{ijf}^{k+2}} q_{ijf} q_{ijf}^T, \quad \left.\frac{\partial \Xi_{ij}}{\partial q_{cd}}\right|_{\bar{q}=\bar{q}_f} = \sigma \frac{2\delta k^2}{\beta_{cdf}^{k+2}} q_{cdf} q_{cdf}^T, \tag{51}$$

where $\beta_{cdf} = 0.5\| q_{cdf} \|^2$, $q_{cdf} = q_{cf} - q_{df}$. With (51), let $\xi \in \mathbb{R}^{nE}$ we have

$$\xi^T \left.\frac{\partial F(\bar{q},\bar{q}_f)}{\partial \bar{q}}\right|_{\bar{q}=\bar{q}_f} \xi \geq \left(1 - \frac{4\delta k^2 En \max(q_{ijfa}^2)}{\min(\beta_{ijf}^{k+2})}\right) \xi^T \xi, \ (i,j) \in \{1,...,N\}, \ i \neq j \tag{52}$$

where q_{ijfa} is the a^{th} element of q_{ijf} . Therefore, for any given constant k if we choose the tuning constant δ such that

$$1 - \frac{4\delta k^2 En \max(q_{ijfa}^2)}{\min(\beta_{ijf}^{k+2})} > 0 \rightarrow \delta < \frac{\min(\beta_{ijf}^{k+2})}{4k^2 En \max(q_{ijfa}^2)}, (i,j) \in \{1,...,N\}, \ i \neq j \tag{53}$$

then the matrix $\partial F(\bar{q},\bar{q}_f)/\partial\bar{q}\big|_{\bar{q}=\bar{q}_f}$ is positive definite, which in turn implies that the equilibrium point $\bar{q} = \bar{q}_f$ is asymptotically stable.

Step 2.2. Proof of $\bar{q} = \bar{q}_c$ *being the unstable/saddle equilibrium point(s):*

The idea is to consider block matrices on the main diagonal of the matrix $\partial F(\bar{q},\bar{q}_f)/\partial\bar{q}\big|_{\bar{q}=\bar{q}_c}$ and show that there exists at least one block matrix whose determinant is negative. Define $H_{ijc} = \partial \Xi_{ij} / \partial q_{ij}\big|_{\bar{q}=\bar{q}_c}$ and let ϕ_a and ϕ_b be the a^{th} and b^{th} elements of q_{ijc}, $(a,b) \in \{1,...,n\}$, $a \neq b$. We form the matrices H_{ijc}^{ab} from the matrix H_{ijc} as follows

$$H_{ijc}^{ab} = \begin{bmatrix} 1 + 2\delta k \Pi_{ijc}\beta_{ijc}^{k-1} + 2\delta k[(k-1)\Pi_{ijc}\beta_{ijc}^{k-2} + 2k/\beta_{ijc}^{k+2}]\phi_a^2 & 2\delta k[(k-1)\Pi_{ijc}\beta_{ijc}^{k-2} + 2k/\beta_{ijc}^{k+2}]\phi_a\phi_b \\ 2\delta k[(k-1)\Pi_{ijc}\beta_{ijc}^{k-2} + 2k/\beta_{ijc}^{k+2}]\phi_a\phi_b & 1 + 2\delta k \Pi_{ijc}\beta_{ijc}^{k-1} + 2\delta k[(k-1)\Pi_{ijc}\beta_{ijc}^{k-2} + 2k/\beta_{ijc}^{k+2}]\phi_b^2 \end{bmatrix} \tag{54}$$

where $\Pi_{ijc} = 1/\beta_{ijf}^{2k} - 1/\beta_{ijc}^{2k}$. The determinant of H_{ijc}^{ab} is given by

$$\det(H_{ijc}^{ab}) = (1 + 2\delta k \Pi_{ijc}\beta_{ijc}^{k-1})\Delta_{ijc}^{ab} \tag{55}$$

where

$$\Delta_{ijc}^{ab} = 1 + 2\delta k \Pi_{ijc}\beta_{ijc}^{k-1} + 2\delta k[(k-1)\Pi_{ijc}\beta_{ijc}^{k-2} + 2k/\beta_{ijc}^{k+2}](\phi_a^2 + \phi_b^2) \tag{56}$$

Let us calculate the sum:

$$\sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \Delta_{ijc}^{ab} = n(n-1) + 2\delta k(n-1)(2(k-1)+n)\beta_{ijc}^{k-1}/\beta_{ijf}^{2k} + 2\delta k(n-1)(2(k+1)-n)/\beta_{ijc}^{k+1} . \tag{57}$$

Since $n > 1$, picking

$$2(k+1) - n > 0 \rightarrow k > n/2 - 1 \tag{58}$$

ensures that $\sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\Delta_{ijc}^{ab}>0$. Therefore, there exists at least one pair $(a,b)\in\{1,...,n\}$ denoted by (a^*,b^*) such that $\Delta_{ijc}^{a^*b^*}>0$. Now for all $(i,j)\in\{1,...,N\},\ i\neq j$ let us consider the sum:

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\det(H_{ijc}^{a^*b^*})}{\Delta_{ijc}^{a^*b^*}}\beta_{ijc}=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(\beta_{ijc}+2\delta k\Pi_{ijc}\beta_{ijc}^{k}). \tag{59}$$

On the other hand, multiplying both sides of $F(\bar{q}_c,\bar{q}_f)=0$ with $\bar{q}_c^T$ results in $\bar{q}_c^T F(\bar{q}_c,\bar{q}_f)=0$, which is expanded to

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(q_{ijc}^T(q_{ijc}-q_{ijf})+2\delta kN\Pi_{ijc}\beta_{ijc}^{k})=0. \tag{60}$$

Substituting (60) into (59) results in

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\det(H_{ijc}^{a^*b^*})}{\Delta_{ijc}^{a^*b^*}}\beta_{ijc}=\frac{1}{N}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(N-2)\beta_{ijc}+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}q_{ijc}^T q_{ijf}. \tag{61}$$

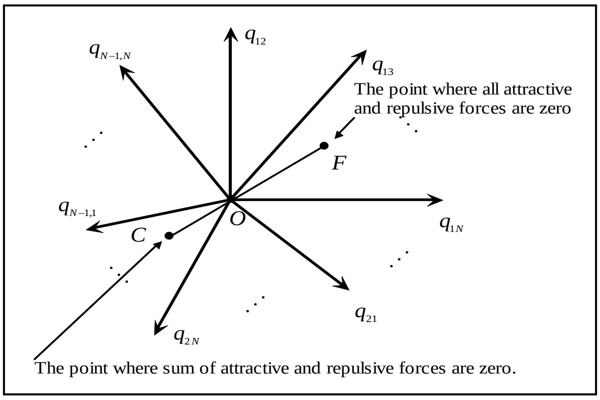

Figure 2. *Illustration of location of critical points*

The term $\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(q_{ijc}^T q_{ijf}\right)$ is strictly negative since at the point where $q_{ij}=q_{ijf}$ (the point F in Figure 2) all attractive and repulsive forces are equal to zero while at the point where $q_{ij}=q_{ijc}$ (the point C in Figure 2) the sum of attractive and repulsive forces is equal to zero (see Section 2 for discussion of a simple case). Therefore the point $q_{ij}=0$ (the point O in Figure 2) must locate between the points $q_{ij}=q_{ijf}$ and $q_{ij}=q_{ijc}$, see Figure 2. Furthermore if we write (60) as

$$2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\beta_{ijc}+\delta kN(\beta_{ijc}^{k}/\beta_{ijf}^{2k}-1/\beta_{ijc}^{k})=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}q_{ijc}^T q_{ijf} \tag{62}$$

we can see that deceasing δ results in decrease in β_{ijc} since β_{ijf} is a bounded constant and the right hand side of (62) is negative. Therefore, choosing a sufficiently small δ ensures that the right hand of (61) is strictly negative since $\beta_{ijc}=0.5\|q_{ijc}\|^2$. That is

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\det(H_{ijc}^{a^*b^*})}{\Delta_{ijc}^{a^*b^*}}\beta_{ijc}<0 \tag{63}$$

which implies that there exists at least one pair $(i^*,j^*)\in\{1,...,N\}$ such that

$$\det(H^{a^*b^*}_{i^*j^*c}) < 0 \,. \quad (64)$$

The inequality (64) implies that at least one eigenvalue of the matrix $\partial F(\bar{q},\bar{q}_f)/\partial\bar{q}\big|_{\bar{q}=\bar{q}_c}$ is negative. This in turn guarantees that $\bar{q}_c$ is unstable/saddle equilibrium point of (47). □

Extension to bounded control

When two or more agents come very closed to each other, the control u_i given in (35) can be very large in magnitude. This is undesired. Hence it is necessary to consider a bounded control law for u_i. Fortunately, this can be easily achieved by replacing the control u_i given in (35) by a bounded control such as

$$u_i = -C\Omega_i \sqrt{1+\sum_{i=1}^{N}\|\Omega_i\|^2} \,. \quad (65)$$

The reason we use $\sqrt{1+\sum_{i=1}^{N}\|\Omega_i\|^2}$ instead of $\sqrt{1+\|\Omega_i\|^2}$ is that the former makes it easy to investigate the dynamics of inter-related agents. Indeed, with the bounded control (65) the derivative of the total potential function φ now becomes (instead of (40)):

$$\dot{\varphi} = -\sum_{i=1}^{N}\Omega_i^T C\Omega_i \Big/ \sqrt{1+\sum_{i=1}^{N}\|\Omega_i\|^2} \,. \quad (66)$$

On the other hand, the dynamics of inter-related agents (46) is changed to

$$\dot{q}_{ij} = -C(\Omega_i - \Omega_j) \Big/ \sqrt{1+\sum_{i=1}^{N}\|\Omega_i\|^2},\ (i,j)\in\{1,...,N\} \,. \quad (67)$$

Stability analysis can be carried out the same lines as in Subsection 3.2. It is noted that with the bounded control, the agents take longer time to approach their desired locations.

CONCLUSIONS

We have presented a constructive method to design controllers that forces a group of N mobile agents to achieve a particular formation in terms of shape, location and orientation while avoiding collisions among themselves. The control development was based on construction of new local potential functions, and guaranteeing that all critical points, besides the desired points in formation, are either saddles or unstable points. Both stabilization and tracking control problems of formation were addressed. Formal analysis of the convergence and feasibility of the control solutions have also been discussed for cases when bounded controls are used. It has been shown that the proposed controller design method can indeed guarantee the convergence of agents to a desired formation, which can either be stationary or moving. A combination of the proposed controllers in this paper with a gradient climbing algorithm 0 could result in potential applications such as search, self-cooperative transportation, and target seeking and attack.

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