SYNTHESIS OF OPTIMAL ADAPTIVE CONTROL SYSTEMS FOR A CLASS OF MIMO LINEAR WITH VARIABLE PARAMETERS SYSTEMS UNDER EXOGENOUS DISTURBANCE

Ngo Tri Nam Cuong[1*], Le Van Chuong[2], Ta Hung Cuong[2], Phung Thi Thanh[3]
[1]Institute of Electrical Engineering and Automation, [2]Vinh University, [3]Industrial University of Vinh

ARTICLE INFO

Received: 04/02/2023
Revised: 27/4/2023
Published: 28/4/2023

KEYWORDS

Automatic control
Adaptive control
Optimal control
Parameter identification
Multiple Input - Multiple Output

ABSTRACT

This paper introduces a control system synthesis method for linear MIMO systems with variable parameters and the impact of unmeasured external disturbances, which is very common in industrial fields. For practical applications, optimal control methods are often used for the system to achieve the desired quality parameters. However, this method is limited because it only guarantees the system's robustness when there are uncertain dynamic components and many external effects. Therefore, with the above class of systems, the article synthesizes control rules based on the combination of optimal control and adaptive control to compensate for uncertain components. The results are adaptive control law compensating for the influence of variable parameter components, external disturbances, and optimal controller for dynamic components with fixed parameters. The article's proposed control system is simple, easy to implement, has high control quality, and ensures optimal ability, adaptability, and good interference resistance. Simulation results on Matlab Simulink software show the correctness and effectiveness of the research results.

TỔNG HỢP HỆ ĐIỀU KHIỂN TỐI ƯU THÍCH NGHI CHO LỚP ĐỐI TƯỢNG MIMO TUYẾN TÍNH CÓ THAM SỐ THAY ĐỔI DƯỚI TÁC ĐỘNG CỦA NHIỄU BÊN NGOÀI

Ngô Trí Nam Cường[1*], Lê Văn Chương[2], Tạ Hùng Cường[2], Phùng Thị Thanh[3]
[1] Viện Kỹ thuật điện và Tự động hóa, [2]Trường Đại học Vinh, [3]Trường Đại học Công nghiệp Vinh

THÔNG TIN BÀI BÁO

Ngày nhận bài: 04/02/2023
Ngày hoàn thiện: 27/4/2023
Ngày đăng: 28/4/2023

TỪ KHÓA

Điều khiển tự động
Điều khiển thích nghi
Điều khiển tối ưu
Nhận dạng tham số
Nhiều đầu vào - nhiều đầu ra

TÓM TẮT

Bài báo giới thiệu phương pháp tổng hợp hệ thống điều khiển cho một lớp đối tượng MIMO tuyến tính có các tham số thay đổi và chịu tác động của nhiễu bên ngoài không đo được, rất thường gặp trong các lĩnh vực công nghiệp. Đối với các ứng dụng thực tế, để hệ thống đạt được những chỉ tiêu chất lượng mong muốn thường sử dụng phương pháp điều khiển tối ưu. Mặc dù vậy, phương pháp này tồn tại hạn chế là không đảm bảo tính bền vững cho hệ thống khi có các thành phần động học bất định và nhiều tác động bên ngoài. Do đó, với lớp các đối tượng nói trên, bài báo thực hiện tổng hợp luật điều khiển trên cơ sở kết hợp giữa điều khiển tối ưu và điều khiển thích nghi bù trừ các thành phần bất định. Các kết quả thu được là luật điều khiển thích nghi bù trừ ảnh hưởng của thành phần tham số động học thay đổi, nhiễu ngoài và bộ điều khiển tối ưu cho thành phần động học có các tham số cố định. Hệ thống điều khiển do bài báo đề xuất đơn giản, dễ thể hiện kỹ thuật, có chất lượng điều khiển cao, đảm bảo khả năng tối ưu, thích nghi và kháng nhiễu tốt. Tính đúng đắn và hiệu quả của các kết quả nghiên cứu được minh chứng bằng mô phỏng trên phần mềm Matlab Simulink.

DOI: https://doi.org/10.34238/tnu-jst.7282

[*] Corresponding author. *Email: ncuong792000@gmail.com*

1. Introduction

Multiple-Input Multiple-Output (MIMO) Linear Systems are commonly encountered in industry, transportation, and energy,... During the working process, the dynamic parameters of the system change due to: the evolution of the load; environment; vibration of the actuator,... In many cases, the system may suffer from unmeasured external disturbances. In response to the increasing requirements for product quality, in the past years, there has been much research on synthesizing control systems for the above plants with good results [1] – [15]. However, some outstanding issues still need to be satisfactorily resolved. In [1] – [4], for the adaptive control algorithm to converge, it is necessary to know the limit of the variable parameter components, but in many cases, this cannot be done. The robust control method based on sliding mode control [5]–[9] has a significant chattering effect when the uncertainty component varies greatly, adversely affecting the system's control quality. Fuzzy logic-based many approaches have been investigated because of their capability to make inferences under uncertainty [10] – [12]. However, synthesizing fuzzy control rules depends on expert knowledge, so applying these controllers to the area without expert knowledge will be difficult. The optimal control method [13] – [15] is only effective when the dynamic parameters of the plant change insignificantly. Besides, the above papers have not mentioned the impact of external disturbances when the system is placed in the actual working environment. Below, the article introduces a method of synthesizing an optimal adaptive controller for a class of MIMO linear systems with variable parameter composition and affected by external disturbances. The control system proposed by this paper has good adaptability, anti-interference ability, and high control quality. The research results are simulated using Matlab Simulink software to demonstrate the correctness and effectiveness of the proposed method.

2. Synthesis of optimal adaptive control systems

Suppose that a class of MIMO linear systems has the equation:

$$\dot{\mathbf{x}} = \left[\mathbf{A} + \mathbf{\Delta A}(t)\right]\mathbf{x} + \left[\mathbf{B} + \mathbf{\Delta B}(t)\right]\mathbf{u} + \mathbf{d}(t), \tag{1}$$

where: $\mathbf{x} = \left[x_1, x_2, ..., x_n\right]^T$ is state vector; $\mathbf{u} = \left[u_1, u_2, ..., u_m\right]^T$ is control vector; $\mathbf{d}(t) = \left[d_1, d_2, ..., d_n\right]^T$ is external disturbance vector, $\sum_{i=1}^{n} |d_i| < d_m$; $\mathbf{A}$, $\mathbf{\Delta A}(t)$ are a pair of matrices of dimension $n \times n$; $\mathbf{A}$ is a matrix with constant elements and is a Hurwitz matrix; $\mathbf{\Delta A}(t)$ is a matrix with elements Δa_{ij} that change uncertainty, $i, j = \overline{1,n}$; $\mathbf{B}$, $\mathbf{\Delta B}(t)$ are a pair of matrices of dimension $n \times m$; $\mathbf{B}$ is a matrix with constant elements; $\mathbf{\Delta B}(t)$ is a matrix with elements Δb_{ij} that change uncertainty, $i = \overline{1,n}$, $j = \overline{1,m}$. It is assumed that: dynamic parameters and external disturbance change slowly and, in the transient process, do not change significantly, i.e. time derivatives $\Delta \dot{a}_{ij} \approx 0$, $\Delta \dot{b}_{ij}(t) \approx 0,$ $\dot{d}_i(t) \approx 0$.

To solve the optimization problem, we first need to define the objective function, which represents the optimal criterion. Starting from the requirements of the control process, we determine the goals to be achieved from the perspective of physical nature and build an objective function that reflects the goals to be achieved. In the control problem, most of the objective functions are chosen as squared state vectors. To clarify the problem, we consider the control plant with the following variable parameters:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{u} . \tag{2}$$

Suppose we choose an objective function of the form:

$$J = \int_{t_0}^{t_f} \mathbf{x}^T \underline{\mathbf{Q}}(t)\mathbf{x}dt, \tag{3}$$

where $\underline{\mathbf{Q}}(t)$ is a positive semi-deterministic matrix.

However, if we only focus on minimizing the objective function (3) without considering other factors, then for the objective function (3) to reach its extreme value, it may require the control function $\mathbf{u}(t)$ to be infinitely large. This leads to a technical impossibility and, of course, no practical significance.

To solve this problem, one adds the squared form of the control vector to the objective function as follows:

$$J = \int_{t_0}^{t_f} \left[\mathbf{x}^T \underline{\mathbf{Q}}(t)\mathbf{x} + \mathbf{u}^T \underline{\mathbf{R}}(t)\mathbf{u}\right]dt, \tag{4}$$

with $\underline{\mathbf{R}}(t)$ is a positive semi-deterministic matrix.

In the case when the endpoint t_f is fixed, the objective function is usually chosen:

$$J = \mathbf{x}^T(t_f)\underline{\mathbf{S}}\mathbf{x}(t_f) + \int_{t_0}^{t_f} \left[\mathbf{x}^T \underline{\mathbf{Q}}(t)\mathbf{x} + \mathbf{u}^T \underline{\mathbf{R}}(t)\mathbf{u}\right]dt, \tag{5}$$

where $\underline{\mathbf{S}}$ is a diagonal matrix with elements $s_{ij} > 0$.

With the plant (2), the objective functional minimization problem (5) is a common optimal control problem and the control laws are known [16]-[18] as follows:

$$\mathbf{u}_{\text{op}} = -\underline{\mathbf{R}}^{-1}(t)\mathbf{B}^T(t)\underline{\mathbf{P}}(t)\mathbf{x}(t), \tag{6}$$

where $\underline{\mathbf{P}}(t)$ is the solution of the Riccati equation:

$$\dot{\underline{\mathbf{P}}}(t) = -\underline{\mathbf{P}}(t)\mathbf{A}(t) - \mathbf{A}^T(t)\underline{\mathbf{P}}(t) - \underline{\mathbf{Q}}(t) + \underline{\mathbf{P}}(t)\mathbf{B}(t)\underline{\mathbf{R}}^{-1}(t)\mathbf{B}^T(t)\underline{\mathbf{P}}(t), \tag{7}$$

with the boundary condition: $\underline{\mathbf{P}}(t_f) = \underline{\mathbf{S}}$. (8)

We see that with control law (6), Riccati equation (7), boundary condition (8), and the optimal criterion (5), the synthesis of the control system for plant (2) will encounter many difficulties to implement in engineering. In particular, this difficulty will increase when the system is affected by external disturbance.

The above complications and problems will be eliminated if matrices $\mathbf{A}$, $\mathbf{B}$, $\underline{\mathbf{Q}}$, $\underline{\mathbf{R}}$ are constant matrices and $t_f = \infty$ [17], [18]. This means that we somehow identify and compensate for the variable parameter components $\mathbf{\Delta A}(t)$, $\mathbf{\Delta B}(t)$ and unmeasured external disturbance $\mathbf{d}(t)$ of plant (1) so that equation (1) becomes:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}. \tag{9}$$

For the plant (9) the known optimal control law [16], [18] is as follows:

$$\mathbf{u}_{\text{op}} = -\underline{\mathbf{R}}^{-1}\mathbf{B}^T\underline{\mathbf{P}}\mathbf{x}, \tag{10}$$

where $\underline{\mathbf{P}}$ is the solution of the Riccati equation:

$$\dot{\underline{\mathbf{P}}} = -\underline{\mathbf{P}}\mathbf{A} - \mathbf{A}^T\underline{\mathbf{P}} - \underline{\mathbf{Q}} + \underline{\mathbf{P}}\mathbf{B}\underline{\mathbf{R}}^{-1}\mathbf{B}^T\underline{\mathbf{P}}. \tag{11}$$

We see that the control law (10) is easy to implement technology. Following this approach, the article builds a control algorithm for the class of system (1) based on optimal and adaptive control. The optimal adaptive control law for class (1) has the form:

$$\mathbf{u} = \mathbf{u}_{\text{op}} + \mathbf{u}_{\text{ad}}, \tag{12}$$

where: $\mathbf{u}_{\text{op}}$ is the optimal control law; $\mathbf{u}_{\text{ad}}$ is an adaptive control law.

The identification model for uncertain parameters in (1) can be written:

$$\dot{\mathbf{x}}_{\mathrm{M}} = \left[\mathbf{A} + \mathbf{\Delta\hat{A}}(t)\right]\mathbf{x}_{\mathrm{M}} + \left[\mathbf{B} + \mathbf{\Delta\hat{B}}(t)\right]\mathbf{u} + \hat{\mathbf{d}}(t), \tag{13}$$

where: $\mathbf{x}_{\mathrm{M}} = \left[x_{\mathrm{M1}}, x_{\mathrm{M2}}, ..., x_{\mathrm{M}n}\right]^T$ is state vector of the model; $\mathbf{\Delta\hat{A}}(t)$ and $\mathbf{\Delta\hat{B}}(t)$ (with the corresponding elements $\Delta\hat{a}_{ij}$ and $\Delta\hat{b}_{ij}$) are the estimated matrices of $\mathbf{\Delta A}(t)$, $\mathbf{\Delta B}(t)$, respectively; $\hat{\mathbf{d}}(t)$ (has elements $\hat{d}_i$) is the estimated vector of $\mathbf{d}(t)$.

Transforming equations (1) and (13), we get the error equation:

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{\Delta\tilde{A}}(t)\mathbf{e} + \mathbf{\Delta\tilde{B}}(t)\mathbf{u} + \tilde{\mathbf{d}}(t), \tag{14}$$

where:

$$\mathbf{e} = \left[e_1, e_2, \ldots, e_n\right]^T = \mathbf{x} - \mathbf{x}_{\mathrm{M}}; \tag{15}$$

$$\mathbf{\Delta\tilde{A}}(t) = \left[\tilde{a}_{ij}\right] = \mathbf{\Delta A}(t) - \mathbf{\Delta\hat{A}}(t); \tag{16}$$

$$\mathbf{\Delta\tilde{B}}(t) = \left[\tilde{b}_{ij}\right] = \mathbf{\Delta B}(t) - \mathbf{\Delta\hat{B}}(t); \tag{17}$$

$$\tilde{\mathbf{d}}(t) = \left[\tilde{d}_i\right] = \mathbf{d}(t) - \hat{\mathbf{d}}(t). \tag{18}$$

The identification process will converge when $\mathbf{\Delta\tilde{A}}(t) \to 0$; $\mathbf{\Delta\tilde{B}}(t) \to 0$; $\tilde{\mathbf{d}}(t) \to 0$. Because $\mathbf{A}$ is a Hurwitz matrix, so $\mathbf{e} \to 0$; in other words, system (14) is stable.

Theorem: *The variable parameter components and unmeasured external disturbance in (1) will be compensated if the adaptive control law* $\mathbf{u}_{\mathrm{ad}}$ *is selected as follows:*

$$\mathbf{u}_{\mathrm{ad}} = -\mathbf{G}\left[\left[\Delta\hat{a}_{ij}\right]\mathbf{x} + \left[\Delta\hat{b}_{ij}\right]\mathbf{u} + \left[\hat{d}_i(t)\right]^T\right], \tag{19}$$

with adaptive update laws:

$$\Delta\hat{a}_{ij} = \int e_j \overline{\mathbf{P}}_i \mathbf{e} dt + \Delta a_{ij}^0;\ i, j = \overline{1, n}; \tag{20}$$

$$\Delta\hat{b}_{ij} = \int u_j \overline{\mathbf{P}}_i \mathbf{e} dt + \Delta b_{ij}^0;\ i = \overline{1, n};\ \ j = \overline{1, m}; \tag{21}$$

$$\hat{d}_i = \int \overline{P}_i \mathbf{e} dt;\ i = \overline{1, n}. \tag{22}$$

where $\mathbf{P}$ *is a positive definite symmetric matrix;* $\overline{\mathbf{P}}_i$ *is the i-th row of the matrix* $\mathbf{P}$*;* $\mathbf{G} = \mathbf{B}^+$, *with* $\mathbf{B}^+$ *is the pseudo-inverse matrix of* $\mathbf{B}$.

Proof:

For equations (14), the Lyapunov function is selected as follows:

$$V = \mathbf{e}^T\mathbf{Pe} + \sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\tilde{a}_{ij}^2 + \sum_{i=1}^{n}\sum_{j=1}^{m}\Delta\tilde{b}_{ij}^2 + \sum_{i=1}^{n}\tilde{d}_i^2, \tag{23}$$

where $\mathbf{P}$ is a positive definite symmetric matrix.

Take the derivative of both sides of the equation (23), we have:

$$\dot{V} = \dot{\mathbf{e}}^T\mathbf{Pe} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} + 2\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\dot{\tilde{a}}_{ij} + 2\sum_{i=1}^{n}\sum_{j=1}^{m}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} + 2\sum_{i=1}^{n}\dot{\tilde{d}}_i\tilde{d}_i. \tag{24}$$

The system (13) will be stable if the derivative of the Lyapunov function $\dot{V} < 0$.

Substituting (14) into (24):

$$\dot{V} = \left[\mathbf{e}^T\mathbf{A}^T + \mathbf{e}^T\mathbf{\Delta\tilde{A}}^T + \mathbf{u}^T\mathbf{\Delta\tilde{B}}^T + \tilde{\mathbf{d}}^T(t)\right]\mathbf{Pe} + \mathbf{e}^T\mathbf{P}\left[\mathbf{eA} + \mathbf{\Delta\tilde{A}e} + \mathbf{\Delta\tilde{B}u} + \tilde{\mathbf{d}}(t)\right] +$$
$$+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\dot{\tilde{a}}_{ij}\Delta\tilde{a}_{ij} + \Delta 2\sum_{i=1}^{n}\sum_{j=1}^{m}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} + 2\sum_{i=1}^{n}\dot{\tilde{d}}_i\tilde{d}_i. \tag{25}$$

Continuing the transformation (25), we have:

$$\dot{V} = \mathbf{e}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e} + 2\mathbf{e}^T\mathbf{\Delta}\tilde{\mathbf{A}}\mathbf{P}\mathbf{e} + 2\mathbf{u}^T\mathbf{\Delta}\tilde{\mathbf{B}}^T\mathbf{P}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}(t) +$$

$$+2\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\dot{\tilde{a}}_{ij}\Delta\tilde{a}_{ij} + 2\sum_{i=1}^{n}\sum_{j=1}^{m}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} + 2\sum_{i=1}^{n}\dot{\tilde{d}}_i\tilde{d}_i. \quad (26)$$

From (26), withdraw the condition for $\dot{V} < 0$ as follows:

$$\mathbf{e}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e} < 0; \quad (27)$$

$$2\mathbf{e}^T\mathbf{\Delta}\tilde{\mathbf{A}}^T\mathbf{P}\mathbf{e} + 2\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\dot{\tilde{a}}_{ij}\Delta\tilde{a} = 0; \quad (28)$$

$$2\mathbf{u}^T\mathbf{\Delta}\tilde{\mathbf{B}}^T\mathbf{P}\mathbf{e} + 2\sum_{i=1}^{n}\sum_{j=1}^{m}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} = 0; \quad (29)$$

$$2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}(t) + 2\sum_{i=1}^{n}\dot{\tilde{d}}_i\tilde{d}_i = 0. \quad (30)$$

Returning to inequality (27), since $\mathbf{A}$ is a Hurwitz matrix, we have:

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}; \quad (31)$$

with $\mathbf{Q}$ is a positive definite matrix.

From (31), we see that condition (27) is completely satisfied.

Next, solving equations from (28) to (30), we have:

$$\Delta\dot{\tilde{a}}_{ij} = -e_j\bar{\mathbf{P}}_i\mathbf{e};\ i, j = \overline{1,n}; \quad (32)$$

$$\Delta\dot{\tilde{b}}_{ij} = -u_j\bar{\mathbf{P}}_i\mathbf{e};\ i = \overline{1,n} \text{ and } j = \overline{1,m}; \quad (33)$$

$$\dot{\tilde{d}}_i = -\bar{\mathbf{P}}_i\mathbf{e};\ i = \overline{1,n}; \quad (34)$$

where $\bar{\mathbf{P}}_i$ is the i-th row of the matrix $\mathbf{P}$.

If simultaneously satisfied from (32) to (34), then the derivative $\dot{V} < 0$, so the system (14) is stable.

From (16) and (32), with the attention that the matrix $\mathbf{\Delta A}(t)$ contains slowly variable elements, i.e. $\Delta\dot{a}_{ij} \approx 0$. Identify law for the uncertain parameters in the matrix $\mathbf{\Delta A}(t)$:

$$\Delta a_{ij} \simeq \Delta\hat{a}_{ij}(t) = \int e_j\bar{\mathbf{P}}_i\mathbf{e}dt + \Delta a_{ij}^0; \quad (35)$$

with $i, j = \overline{1,n}$; Δa_{ij}^0 is initialization value.

From (17) and (33), with the attention that the matrix $\mathbf{\Delta B}(t)$ contains slowly variable elements, i.e. $\Delta\dot{b}_{ij}(t) \approx 0$. Identify law for the uncertain parameters in the matrix $\mathbf{\Delta B}(t)$:

$$\Delta b_{ij} \simeq \Delta\hat{b}_{ij} = \int u_j\bar{\mathbf{P}}_i\mathbf{e}dt + \Delta b_{ij}^0; \quad (36)$$

with $i = \overline{1,n}$; $j = \overline{1,m}$; and Δb_{ij}^0 is initialization value.

From (18) and (34), because of slow-varying external disturbance $\dot{d}_i(t) \approx 0$. The unmeasurable external disturbance vector identification law $\mathbf{d}(t)$ has the following elements:

$$\dot{\hat{d}}_i(t) = \bar{\mathbf{P}}_i\mathbf{e} \quad \rightarrow \quad d_i(t) \simeq \hat{d}_i(t) = \int \bar{\mathbf{P}}_i\mathbf{e}dt;\ i = \overline{1,n}. \quad (37)$$

Next, based on the recognition algorithms, we build an adaptive control law $\mathbf{u}_{ad}$ to compensate for the influence of the variable parameter components and external disturbance.

We set:

$$\mathbf{f}(t) = \mathbf{\Delta A}(t)\mathbf{x} + \mathbf{\Delta B}(t)\mathbf{u} + \mathbf{d}(t), \tag{38}$$

with $\mathbf{f}(t) = \left[f_1(t), f_2(t), ..., f_n(t) \right]^T$.

From (38), we rewrite (1) to:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{If}(t), \tag{39}$$

where $\mathbf{I} \in \mathbb{R}^{n \times n}$ has main diagonal elements, $I_{ij} = 1$ are rows which corresponds to the vector $\mathbf{f}(t)$ in the case $\left| f_i(t) \right| \neq 0$; other elements $I_{ij} = 0$ in the case $i \neq j$ and $\left| f_i(t) \right| = 0, \left(i, j = \overline{1, n} \right)$.

Substituting (12) into (39), we have:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}_{\text{op}} + \mathbf{Bu}_{\text{ad}} + \mathbf{If}(t). \tag{40}$$

From the results of identification from (35) to (37), attention (38), and combination (40), we choose:

$$\mathbf{u}_{\text{ad}} = -\mathbf{G}\hat{\mathbf{f}}(t). \tag{41}$$

where $\mathbf{G}$ is the gain matrix; $\hat{\mathbf{f}}(t) = \left[\Delta\hat{a}_{ij} \right]\mathbf{x} + \left[\Delta\hat{b}_{ij} \right]\mathbf{u} + \left[\hat{d}_i(t) \right]^T$.

Uncertainty components will be compensated by bringing (41) to (40) if:

$$-\mathbf{BG}\hat{\mathbf{f}}(t) + \mathbf{If}(t) = 0. \tag{42}$$

To satisfy (42) means:

$$\mathbf{BG} = \mathbf{I}. \tag{43}$$

From (43), we choose $\mathbf{G} = \mathbf{B}^+$, where $\mathbf{B}^+$ is the pseudo-inverse matrix of $\mathbf{B}$ [19].

With the control vector $\mathbf{u}_{\text{ad}}$ (41) fed to the input of plant (1), the variable parameter components and external disturbance are compensated, and then (1) takes the form:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}_{\text{op}}. \tag{44}$$

Thus, the expressions (35), (36), (37) with the adaptive law (41) are fed to the input of the plant (1) to compensate for the variable parameter components and external disturbance, and then (1) becomes (44).

The theorem is proven.

For (44), the optimal control law is known in (10). From there, the optimal-adaptive control law for plant (1) is an expression (12) in which $\mathbf{u}_{\text{op}}$ and $\mathbf{u}_{\text{ad}}$ are expressed in (10) and (41).

Thus, the article has synthesized the optimal-adaptive law for the class of MIMO linear systems (1); the system has optimal properties, good adaptability, and highanti-interference ability.

3. Results and discussion

Assume that the control object is represented in the form of equation (1) with the dynamic parameter matrices and the external disturbances vector as follows:

$$\begin{aligned} &\mathbf{A} = \begin{bmatrix} -4.0626 & 0.0847 \\ 2.5560 & -4.5329 \end{bmatrix}; \quad \mathbf{\Delta A} = \begin{bmatrix} -1.0157 & 0.0212 \\ 0.6390 & -1.1332 \end{bmatrix} \sin(0.25t); \\ &\mathbf{B} = \begin{bmatrix} 0.7624 & 0.6118 \\ 0.7878 & 0.4936 \end{bmatrix}; \quad \mathbf{\Delta B} = \begin{bmatrix} 0.1906 & 0.1530 \\ 0.1970 & 0.1234 \end{bmatrix} \sin(0.25t); \\ &\mathbf{d}(t) = \begin{bmatrix} 0.35\sin(1.2t) + 0.2\sin(0.5t + 1.5) \\ 0.50\cos(1.0t + 2.0) + 0.3\sin(0.8t) \end{bmatrix}. \end{aligned} \tag{45}$$

Simulation is performed on Matlab Simulink software. The results of identifying variable parameter components and external disturbances $\mathbf{f}(t)$ (38) using the expressions (20), (21), and

(22) of the Theorem are shown in Figure 1. Using controller (12) with $\mathbf{u}_{\text{op}}$, $\mathbf{u}_{\text{ad}}$ shown in (10) and (19), the results of compensating for the effects of variable parameter components and external disturbances are shown in Figure 2, the result of the state vector of the system following the desired set signal vector $\mathbf{x}_{\text{d}} = \left[1.0(t) \quad 2.5(t)\right]$ is shown in Figure 3.

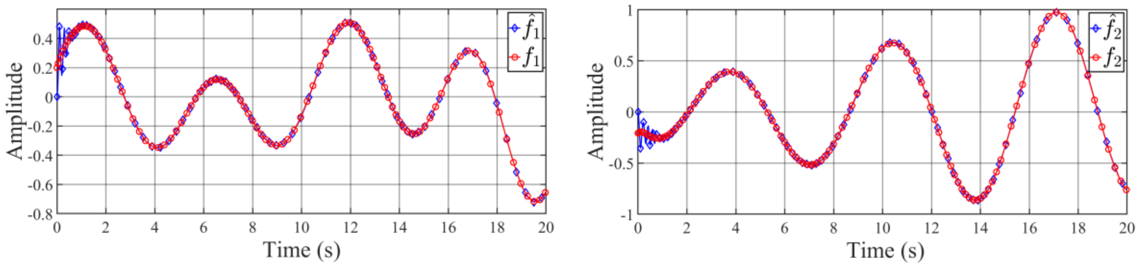


Figure 1. *The results of identifying variable parameter components and external disturbances* $\mathbf{f}(t)$

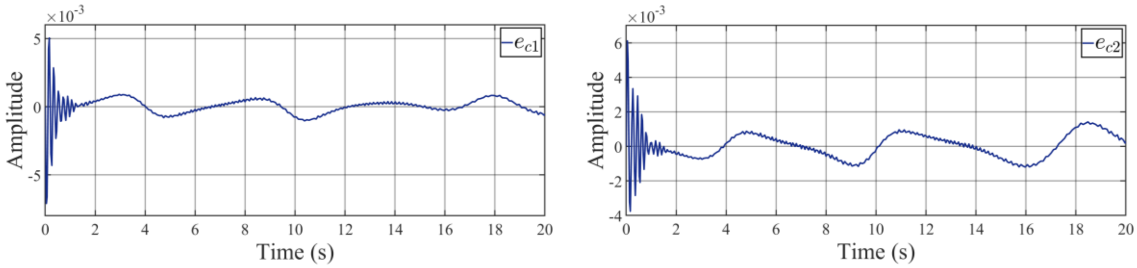
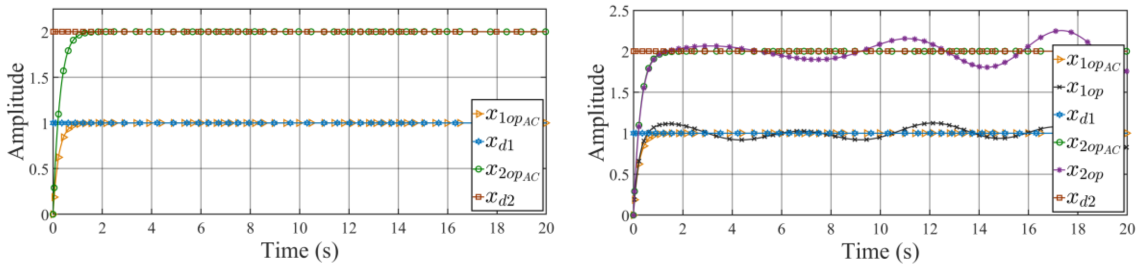


Figure 2. *The error between plant (1) after compensating for variable parameter components, external disturbances by the adaptive control law* $\mathbf{u}_{\text{ad}}$ *(19), and the model (9)*



(a) *(b)*

Figure 3. *(a) Responses of the system for the desired signals* $\mathbf{x}_{\text{d}} = \left[1.0(t) \quad 2.5(t)\right]$;
(b) Compare the system's response when using the optimal-adaptive controller proposed by the article ($\mathbf{x}_{\text{op-ad}}$) *and the normally optimal controller* ($\mathbf{x}_{\text{op}}$)

Figure 1 shows that the variable parameter components and external disturbances have been identified with high accuracy according to the proposed algorithm. From the identification results, performing compensation by adaptive control law (19) makes the system (1) become the system (19) with a small error $e_{\text{c}} \approx 10^{-3}$ shown in Figure 2. The optimal control law (10) has made the system track to the desired set signal $\mathbf{x}_{\text{d}}$ shown in Figure 3 (a) with guaranteed control quality. The advantages of the adaptive optimal control system proposed by the article are shown through comparison with the normally optimal control system with simulation results shown in Figure 3 (b). This comparison shows the high efficiency of the control algorithms proposed by the article for the class of MIMO linear systems with variable parameter components, having the impact of unmeasurable external disturbances, and overcoming the shortcomings pointed out in [1] – [15].

4. Conclusion

This paper has developed a method to synthesize an optimal-adaptive control system for a class of MIMO linear systems with variable parameter components and unmeasurable external disturbances. The theorem was proposed and proved to ensure that the system is invariant with uncertain components for the class MIMO system with equation (1). The control laws for the class of system (1) are synthesized with a combination of optimal control and adaptive control. The proposed article control law is simple and easy to implement in engineering; it ensures the system has high control quality and has good adaptability, optimization, and anti-interference ability. The simulation results once again proved the correctness and effectiveness of the proposed method.

REFERENCES

[1] M. S. Chen and J. M. Wu, “A new model reference adaptive control for linear time-varying systems,” *International journal of adaptive control and signal processing*, vol. 14, no. 4, pp. 469-479, 2000.

[2] G. H. Yang and D. Ye, “Adaptive robust control synthesis for linear systems with time-varying uncertainties,” *IEEE Conference on Industrial Electronics and Applications*, 2007, pp. 1727-1730.

[3] A. K. Chakrabarty and S. Bhattacharya, “Lyapunov based two-stage robust model reference adaptive controller for linear plants with time varying bounded uncertainties,” *IFAC-PapersOnLine*, vol. 49, no. 1, pp. 213-218, 2016.

[4] C. H. Xie and G. H. Yang, “Data-based fault-tolerant control for uncertain linear systems with actuator faults,” *IET Control Theory & Applications*, vol. 10, no. 3, pp. 265-272, 2016.

[6] C. W. Tao, W. Y. Wang, and M. L. Chan, “Design of sliding mode controllers for bilinear systems with time varying uncertainties,” *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 1, pp. 639-645, 2004.

[7] M. Das and C. Mahanta, “Optimal sliding mode controller for systems with mismatched uncertainty,” *IEEE 8th Conference on Industrial Electronics and Applications (ICIEA)*, 2013, pp. 352-357.

[8] M. L. Chan, C. W. Tao, and T. T. Lee, “Sliding mode controller for linear systems with mismatched time-varying uncertainties,” *Journal of the Franklin Institute*, vol. 337, no. 2-3, pp. 105-115, 2000.

[9] K. D. Young and Ü. Özgüner, “Sliding-mode design for robust linear optimal control,” *Automatica*, vol. 33, no. 7, pp. 1313-1323, 1997.

[10] C. W. Tao, M. L. Chan, and T. T. Lee, “Adaptive fuzzy sliding mode controller for linear systems with mismatched time-varying uncertainties,” *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 33, no. 2, pp. 283-294, 2003.

[11] C. C. Weng and W. S. Yu, “Adaptive fuzzy sliding mode control for linear time-varying uncertain systems,” in *IEEE International Conference on Fuzzy Systems (IEEE World Congress on Computational Intelligence)*, 2008, pp. 1483-1490.

[12] C. W. Tao, J. S. Taur, and M. L. Chan, “Adaptive fuzzy terminal sliding mode controller for linear systems with mismatched time-varying uncertainties,” *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 1, pp. 255-262, 2004.

[13] Z. Zhang, S. Zhang, H. Li, and W. Yan, “Cooperative robust optimal control of uncertain multi-agent systems,” *Journal of the Franklin Institute*, vol. 357, no. 14, pp. 9467-9483, 2020.

[14] M. Das and C. Mahanta, “Optimal adaptive sliding mode controller for linear uncertain systems,” *IEEE International Conference on Signal Processing, Computing and Control (ISPCC*), 2013, pp. 1-5.

[15] H. Tan, S. Shu, and F. Lin, “An optimal control approach to robust tracking of linear systems,” *International Journal of Control*, vol. 82, no. 3, pp. 525-540, 2009.

[16] P. Andrew, *Sage and Chelsea C. White, Optimum systems control*. Prentice Hall, Inc. 1977.

[17] M. Athans and P. L. Falb, *Optimal Control*. McGraw-Hill, New York, 1966.

[18] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal control.* John Wiley & Sons, 2012.

[19] J. M. Ortega, *Matrix Theory.* Plenum Press 1987.