BALANCED POSITIVE-REAL TRUNCATION FOR UNSTABLE SYSTEMS: A NOVEL ALGORITHM FOR MODEL REDUCTION IN HIGH-ORDER ELECTRICAL AND ELECTRONIC CIRCUITS

Nguyen Thuy Dung[1], Nguyen Thanh Tung[1*], Dao Huy Du[2], Vu Ngoc Kien[2]
[1]TNU - University of Information and Communication Technology, [2]TNU - University of Technology

ARTICLE INFO

Received: 07/10/2023
Revised: 06/11/2023
Published: 06/11/2023

KEYWORDS

Model Order Reduction
Passivity Preservation
Unstable Systems
Balanced Truncation
Electrical Circuits

ABSTRACT

This article presents a novel algorithm, named BPRU, designed to reduce the complexity of high-order systems while preserving passive (real-positive) properties and stability. Additionally, this technique can also reduce the order of unstable systems. The approach is grounded in solving real-positive Riccati equations, and Riccati H-Infinity equations, combined with matrix analysis techniques like Cholesky decomposition and Singular Value Decomposition (SVD). These methods transform the system into an energy-balanced state from the control and observation Gramians. MATLAB simulations demonstrate that BPRU maintains the crucial physical properties of electrical and electronic circuits and provides effective model order reduction for systems containing both stable and unstable components. Compared to foundational methods such as Positive Real Balanced truncation (PRR) and H-Infinity Balanced truncation (HBR), BPRU yields significantly lower errors, closely matching the original system's transient and frequency responses. This underscores the efficiency and potential of BPRU in reducing model order for electrical and electronic systems, opening up avenues for research in related applications in this field.

CẮT NGẮN CÂN BẰNG THỰC-DƯƠNG CHO HỆ THỐNG KHÔNG ỔN ĐỊNH: MỘT THUẬT TOÁN MỚI ĐỂ RÚT GỌN MÔ HÌNH CHO MẠCH ĐIỆN, ĐIỆN TỬ BẬC CAO

Nguyễn Thuỳ Dung[1], Nguyễn Thanh Tùng[1*], Đào Huy Du[2], Vũ Ngọc Kiên[2]
[1]Trường Đại học Công nghệ Thông tin và Truyền thông – ĐH Thái Nguyên
[2]Trường Đại học Kỹ thuật Công nghiệp – ĐH Thái Nguyên

THÔNG TIN BÀI BÁO

Ngày nhận bài: 07/10/2023
Ngày hoàn thiện: 06/11/2023
Ngày đăng: 06/11/2023

TỪ KHÓA

Giảm bậc mô hình
Bảo toàn sự thụ động
Hệ thống không ổn định
Cắt ngắn cân bằng
Mạch điện

TÓM TẮT

Bài báo giới thiệu một thuật toán mới, mang tên BPRU có khả năng giảm sự phức tạp cho hệ bậc cao, bảo toàn tính chất thụ động (thực dương) và ổn định, đồng thời kỹ thuật này có thể giảm bậc trực tiếp cho các hệ không ổn định. Kỹ thuật này dựa trên việc giải phương trình Riccati thực dương, Riccati H-Infinity, kết hợp với các phép phân tích ma trận Cholesky, SVD để đưa hệ về trạng thái cân bằng năng lượng từ Gramians điều khiển và quan sát của hệ. Kết quả mô phỏng trên Matlab cho thấy BPRU không chỉ duy trì tính chất vật lý quan trọng của mạch điện, điện tử, mà còn cung cấp sự giảm bậc hiệu quả cho hệ chứa cả thành phần ổn định và không ổn định. So sánh với các phương pháp nền tảng là Cắt ngắn cân bằng thực dương (PRR) và Chặt cân bằng H-inf (HBR), BPRU cho kết quả rút gọn mô hình với sai số thấp hơn đáng kể, có đáp ứng xung và đáp ứng tần số bám sát hệ gốc. Điều này cho thấy tính hiệu quả và tiềm năng của BPRU trong việc giảm bậc mô hình cho các hệ thống điện, điện tử, mở ra cơ hội nghiên cứu về các ứng dụng liên quan đến lĩnh vực này.

DOI: https://doi.org/10.34238/tnu-jst.8930

[*] Corresponding author. *Email: nttung@ictu.edu.vn*

1. Introduction

Modern electrical systems have grown remarkably complex and interconnected across vast regions. With thousands of devices and numerous state variables, utilizing detailed models for dynamic analysis becomes impractical. Model Order Reduction (MOR) aims to create low-dimensional systems with similar responses to high-order counterparts, simplifying computation and deployment. Preserving system characteristics like passivity and stability is crucial.

Passivity, the ability to store and transmit energy without an external supply, is a key feature in electrical circuits. Maintaining passivity during reduction is vital. The Frequency-Weighted Positive Real Balanced Truncation technique [1] integrates frequency-weighted reduction with Positive Real-Truncated Balanced Realization to generate passive reduced models. A method in [2] focuses on reducing positive real and bounded real systems while retaining both properties. Salehi et al. [3] propose a phase-enhanced technique using Balanced Truncation and passivity-preserving Grampians via Riccati equations. Stability is preserved via Balanced Truncation in various contexts. The article [3] suggests a phase-enhanced technique that utilizes Balanced Truncation and passivity-preserving Gramians through Riccati equations. Stability is maintained through Balanced Truncation in different situations. The document [4] employs a combination of techniques to reduce the order in a DC-DC converter model while preserving essential properties and stability. The authors in [5] achieve power system reduction with a focus on preserving stability. The research [6] applies Balanced Truncation to improve fault feature extraction in power grids. For unstable systems, direct reduction involves separating them into stable and unstable components. For unstable systems, direct reduction involves decomposing into stable and unstable components. Approaches like H-infinity Balanced Truncation [7] or Linear Quadratic Gaussian Balanced Truncation [8] can directly reduce unstable subsystems.

In this study, we introduce a novel algorithm that effectively preserves passivity and stability while directly reducing the order of unstable systems. We validate the algorithm through simulations on a high-order unstable electrical system using Matlab, comparing its efficacy in reducing both stable and unstable subsystems while maintaining desired characteristics.

2. Materials and Methods

This section introduces a novel algorithm applicable to systems that are both positive (passive) and can be stable or unstable. This model reduction method can reduce the order of stable systems similar to Gramians-based algorithms while also directly reducing the order of unstable systems, all the while preserving the passive (positive) property of the original system. This technique employs a positive-definite Riccati equation (1) and a Riccati equation (2). We refer to this as the Balanced Positive-Real Truncated Realization for Unstable systems (BPRU) algorithm.

Algorithm. Balanced Positive-Real Truncated Realization for Stable and Unstable Systems (BPRU) algorithm:

Input: The Linear Time-Invariant (LTI) system G(s) is potentially stable or unstable and is characterized as minimal, positive real (passive), with an order of *n*. The objective is to reduce the system order to *r* (*r* < *n*).

- Step 1: Solve (1) and (2):

$$\left(\mathbf{B}-\mathbf{X_p C^T}\right)\left(\mathbf{D}+\mathbf{D^T}\right)^{-1}\left(\mathbf{B}-\mathbf{X_p C^T}\right)^{\mathbf{T}} = -\mathbf{X_p A^T} - \mathbf{A X_p} \tag{1}$$

$$\mathbf{A^T Y_{inf}} + \mathbf{Y_{inf} A} + \mathbf{C^T C} = \left(\mathbf{1}-\gamma^{-2}\right)\left(\mathbf{B^T Y_{inf}} + \mathbf{D^T C}\right)^{\mathbf{T}}\left(\mathbf{I}+\mathbf{D^T D}\right)^{-1}\left(\mathbf{B^T Y_{inf}} + \mathbf{D^T C}\right) \tag{2}$$

- Step 2: Cholesky decomposition:

$$\mathbf{X_p} = \mathbf{R_p R_p^*} \tag{3}$$

$$\mathbf{Y_{inf}} = \mathbf{L_{inf} L_{inf}^*} \tag{4}$$

- Step 3: Singular Value Decomposition:

$$\mathbf{L_{inf}}^{\bullet}\mathbf{R_p} = \mathbf{U_{PR}}\mathbf{\Sigma_{PR}}\mathbf{V_{PR}}^{\bullet} \tag{5}$$

- Step 4: Calculate the transformation matrix:

$$\mathbf{T} = \mathbf{R_p}\mathbf{V_{Pr}}\mathbf{\Sigma_{Pr}}^{-\frac{1}{2}} \tag{6}$$

$$\mathbf{T}^{-1} = \mathbf{\Sigma_{Pr}}^{-\frac{1}{2}}\mathbf{U_{Pr}}^{\bullet}\mathbf{L}^{\bullet}_{\mathbf{inf}} \tag{7}$$

- Step 5: Convert to a balanced system:

$$\mathbf{T}^{-1}\mathbf{A}\mathbf{T} = \begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} \end{bmatrix} \tag{8}$$

$$\mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} \mathbf{B_1} \\ \mathbf{B_2} \end{bmatrix} \tag{9}$$

$$\mathbf{CT} = \begin{bmatrix} \mathbf{C_1} & \mathbf{C_2} \end{bmatrix} \tag{10}$$

where: $A_{11} \in R^{r\times r}, A_{12} \in R^{r\times(n-r)}, A_{21} \in R^{(n-r)\times r}, A_{22} \in R^{(n-r)\times(n-r)}, B_1 \in R^{r\times m}, B_2 \in R^{(n-r)\times m}, C_1 \in R^{p\times r}, C_2 \in R^{p\times(n-r)}$

- Step 6: Choose the order r to reduce *r (r < n)*

Output: Reduced-order system with order *r* preserves positive real properties*:*

$$\left(\mathbf{A_r}, \mathbf{B_r}, \mathbf{C_r}, \mathbf{D_r}\right) = (\mathbf{A_{11}}, \mathbf{B_1}, \mathbf{C_1}, \mathbf{D}), \text{ where } A_r \in R^{r\times r}, B_r \in R^{r\times m}, C_r \in R^{p\times r}, D_r \in R^{p\times m}$$

3. Results and Discussion

Consider a dynamical system describing an electrical circuit, modeled by the following matrices.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 6 & -6 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & -6 & 0 & 0 & -6 \\ -6 & 0 & 0 & -10 & 0 & 6 \\ 6 & -6 & 0 & 0 & -6 & 0 \\ 0 & 0 & 6 & -6 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{C}^{\bullet} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{D} = [0.01]$$

We proceed to implement the PRR, HBR, and the proposed BPRU algorithms using MATLAB. Subsequently, we apply each method to gradually reduce the order of the original system from order 6 to order 1. This process yields a plot depicting the absolute errors between the original and reduced-order systems as shown in Table 1.

Table 1. *Comparison of Model Reduction Algorithms*

Reduced Order (r)	PRR Error	HBR Error	BPRU Error
1	0.12736	0.12861	0.12768
2	0.09617	0.11163	0.15010
3	0.07791	0.83380	0.02505
4	0.09741	0.05815	0.01453
5	6.69732	0.06217	0.00551

Table 1 clearly compares highlights the performance differences among the three model reduction algorithms. The BPRU method stands out with consistently low errors, suggestive of its reliability in accurately representing the original system. HBR performs reasonably well but exhibits some fluctuations in error. PRR shows limitations, especially for higher-order reductions, where it yields notable high errors. Therefore, when accuracy is paramount, the BPRU algorithm appears to be the most effective choice among these three algorithms for model reduction in this context.

Analyzing Table 1 reveals distinct patterns in the errors obtained using the different reduction methods. For the PRR method, the most significant error, approximately 6.69732, occurs when reducing the system to order 5. The most accurate reduction, with an error of 0.07791, is achieved at order 3. Contrarily, for the HBR technique, the highest error of 0.83380 emerges at order 3, while the most precise reduction, yielding an error of 0.05815, is realized at order 4. When applying the BPRU algorithm, the error distribution presents unique characteristics. The largest error, approx-imately 0.15010, is recorded at order 2, contrasting sharply with the most optimal performance at order 5, which yields a minimum error of approximately 0.00551.

- The PRR algorithm exhibits varying errors across different reduced orders. The error spikes significantly at r = 5, suggesting that PRR might struggle with maintaining accuracy for higher-order reductions. The errors for other orders are reasonable but not consistently low.

- HBR displays relatively low errors for most of the reduced orders, except at r = 3 where the error spikes significantly. This indicates that HBR is capable of providing accurate results but may have limitations for certain reduced orders.

- The BPRU algorithm consistently produces low errors across all reduced orders. As the reduced order increases, the error decreases, indicating its effectiveness in capturing system dynamics accurately.

Figure 1 shows the step response comparison between the original system and the reduced-order systems using the PRR, HBR, and the proposed BPRU methods. Figure 2 displays the Bode frequency response (Phase, Magnitude) comparison between the original system and the reduced-order systems using the PRR, HBR, and BPRU techniques.

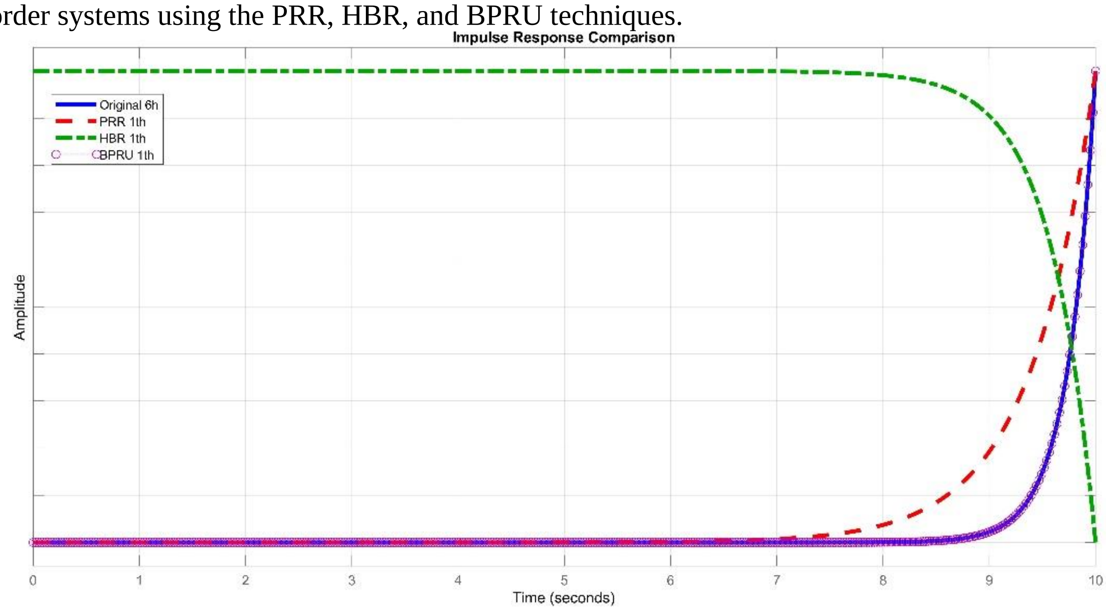

Figure 1. *Compare the impulse response of a first-order reduced system using PRR, HBR, BPRU, and the Original System*

From Figure 1, the following observations can be made:

- The reduced-order systems obtained using the PRR and HBR methods for order 1 exhibit widely different step response behaviors compared to the original system. In contrast, the BPRU technique yields a step response that closely matches the original system's response across the entire time domain.

- The effectiveness of reducing the original system to order 1 is evident. It demonstrates that the reduced-order system of order 1 can be used as a replacement for the original system of order 6 within the time domain when employing the novel BPRU algorithm.

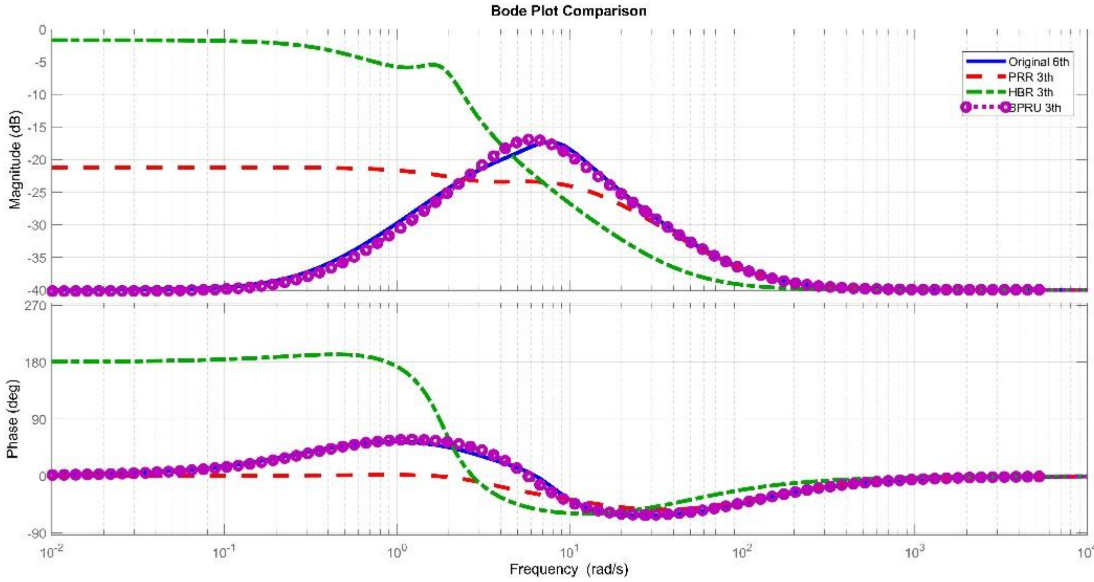

Figure 2. *Bode plot comparing the 3rd order reduction system using PRR, SBR, BPRU, and the Original System*

From Figure 2, the following insights can be drawn:

- The reduced-order systems of order 3 using the PRR and HBR methods result in significant Phase and Magnitude response discrepancies compared to the original system, particularly in the frequency range below 10^3 rad/s. Beyond 10^3 rad/s, these responses converge closely to the original system. Thus, the reduced-order system of order 3 could potentially replace the original system for frequencies greater than 10^3 rad/s. If preserving passivity is the priority, the PRR method is suitable. Conversely, if directly reducing an unstable system is desired, the HBR technique is appropriate.

- Meanwhile, the BPRU algorithm produces frequency responses that match the original system across the entire frequency range. Given the effective order reduction to order 3, the reduced-order system could replace the original system for all frequency ranges. The BPRU algorithm showcases the advantages of both passive and unstable system reduction techniques.

The reduced-order system obtained using the proposed BPRU algorithm provides smaller errors compared to the foundational PRR and HBR methods. It exhibits accurate step and frequency responses closely matching the original system. Moreover, BPRU offers the advantage of effective model reduction while maintaining passivity and avoiding the need to separate the system into stable and unstable subsystems before conducting the reduction.

4. Conclusion

In this study, we introduced a novel algorithm, the Balanced Positive-Real Truncated Realization for Unstable Systems (BPRU), designed to effectively preserve passivity and stability while directly reducing the order of unstable systems. Through simulations on a high-order unstable electrical system and a comprehensive comparison with existing model reduction techniques, we have demonstrated the efficacy of the BPRU algorithm in reducing both stable and unstable subsystems while maintaining desired characteristics.

The results and discussions clearly highlight the strengths of the BPRU algorithm. It consistently produces low errors across different reduced orders, making it a robust choice for model reduction in contexts where accuracy is paramount. The step and frequency responses of the reduced-order systems closely match those of the original system, across both the time and frequency domains. This implies that the BPRU algorithm not only reduces model complexity but also accurately represents the behavior of the original system. Furthermore, the BPRU algorithm offers a distinct advantage in handling unstable systems without the need for separating them into stable and unstable components before reduction, simplifying the reduction process. This feature makes it a valuable tool in dealing with complex electrical systems with both passive and unstable elements.

In conclusion, the BPRU algorithm presents a promising solution for model order reduction in the realm of electrical systems, offering a powerful and efficient approach to preserve system characteristics, such as passivity and stability, while reducing complexity. Its performance, as demonstrated in this study, makes it a compelling choice for engineers and researchers seeking accurate and efficient model reduction techniques for both stable and unstable systems in the field of electrical engineering.

Acknowledgment

This research result is the product of a university-level scientific research project with code ĐH2023-TN07-01, funded by the Thai Nguyen University of Information and Communication Technology (ICTU).

REFERENCES

[1] D. Kumar, U. Zulfiqar, V. Sreeram, M. Imran, W. M. W. Muda, A. Jazlan, and A.-G. Wu, "Positive-Real Truncated Balanced Realization based Frequency-Weighted Model reduction," *2019 Australian & New Zealand Control Conference (ANZCC)*, Auckland, New Zealand, 2019, pp. 145-147.

[2] Z. Salehi, P. Karimaghaee, and M. -H. Khooban, "Mixed Positive-Bounded Balanced Truncation," in *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 7, pp. 2488-2492, July 2021.

[3] Z. Salehi, P. Karimaghaee, and M. -H. Khooban, "A New Passivity Preserving Model Order Reduction Method: Conic Positive Real Balanced Truncation Method," in *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 5, pp. 2945-2953, May 2022.

[4] P. S. Deb, G. Leena, A. K. Vishwakarma, V. Gaur, Tushar, and T. Raghav, "Model Reduction and Comparative Analysis of DC-DC Converter," *2023 3rd International Conference on Advance Computing and Innovative Technologies in Engineering (ICACITE)*, Greater Noida, India, 2023, pp. 115-119.

[5] P. S. Deb and G. Leena, "Model Order Reduction of Single Machine Infinite Bus Power System," *2023 3rd International Conference on Advance Computing and Innovative Technologies in Engineering (ICACITE)*, Greater Noida, India, 2023, pp. 492-496.

[6] J. Han, Z. Zhou, Q. Chen, F. Wang, and X. Wang, "Formulation of Fault Characteristics Based on Model Reduction in High Penetration Electronics System," *2023 5th Asia Energy and Electrical Engineering Symposium (AEEES)*, Chengdu, China, 2023, pp. 521-526.

[7] D. Mustafa, "Reduced-order robust controllers: H/sub infinity /-balanced truncation and optimal projection," *29th IEEE Conference on Decision and Control*, Honolulu, HI, USA, vol.2, pp. 488-493, 1990.

[8] R. W. H. Merks, M. Mirzakhalili, and S. Weiland, "On the Non-optimality of Linear Quadratic Gaussian Balanced Truncation for Constrained Order Controller Design," *2019 18th European Control Conference (ECC)*, Naples, Italy, 2019.